Section 6.6 **The convolution integral.**

Theorem. If $F(s) = \mathcal{L}\{f(t)\}$ and $G(s) = \mathcal{L}\{g(t)\}$ both exist for $s > a \geq 0$, then

$$H(s) = F(s)G(s) = \mathcal{L}\{h(t)\}, \qquad s > a,$$

where

$$h(t) = \int_0^1 f(t-\tau)g(\tau)d\tau = \int_0^1 f(\tau)g(t-\tau)d\tau.$$

The function h is known as a **convolution** of f and g ($h(t) = (f * g)(t)$); the integrals in the formula for $h(t)$ are known as **convolution integrals**.

Properties of the convolution.

1. $f * g = g * f$
2. $f * (g_1 + g_2) = f * g_1 + f * g_2$
3. $(f * g) * h = f * (g * h)$
4. $f * 0 = 0 * f = 0$

Example 1. Find the Laplace transform of the function

$$f(t) = \int_0^1 (t-\tau)^2 \cos 2\tau \; d\tau$$

Example 2. Find $\mathcal{L}^{-1}\left\{\frac{1}{s^4(s^2+1)}\right\}$

Example 3. Express the solution of the initial value problem

$$y'' + \omega^2 y = g(t), \qquad y(0) = 0, y'(0) = 1$$

in terms of a convolution integral.