**MATH 308 – 201** **Practice Problems for Exam 2** **Summer 2018**

over sections 3.5 – 3.8, 6.1 – 6.6, 7.1 – 7.3

1. Find the general solution of the equation/solve the initial value problem
   (a) $y'' + 6y' + 9y = t\cos(2t)$
   (b) $4y'' + y' = 4x^3 + 48x^2 + 1$
   (c) $y'' + 2y' + y = 4e^{-t}$, $y(0) = 2$, $y'(0) = 1$

2. Find the general solution of the equation $y'' + 6y' + 9y = \dfrac{e^{-3x}}{1+2x}$

3. A spring is stretch 10 cm by a force of 3 N. A mass of 2 kg is hung from the spring and is also attached to a viscous damper that exerts a force of 3 N when the velocity of the mass 5 m/s. If the mass is pulled down 5 cm below its equilibrium position and given an initial velocity of 10 cm/s, determine its position $u$ at any time. Find the quasifrequency of the motion.

4. A mass weighting 8 lb is attached to a spring hanging from the ceiling and comes to rest at its equilibrium position. At $t = 0$, an external force $F(t) = 2\cos 2t$ lb is applied to the system. If the spring constant is 10 lb/ft and the damping constant is 1 lb-sec/ft, find the steady-state solution for the system. What is the resonance frequency for the system?

5. A mass weighing 4 lb stretches a spring 1.5 in. The mass is given a positive displacement 2 in from its equilibrium position and released with no initial velocity. Assuming that there is no damping and the mass is acted on by an external force of $2\cos 3t$ lb,
   (a) Formulate the initial value problem describing the motion of mass
   (b) Solve the initial value problem.
   (c) If the given external force is replaced by a force $4\cos\omega t$ of frequency $\omega$, find the value of $\omega$ for which resonance occurs.

6. Find the Laplace transform of the given function.
   (a) $f(t) = \begin{cases} \dfrac{t}{2}, & 0 \le t < 6 \\ 3, & t \ge 6 \end{cases}$
   (b) $f(t) = (t^2 - 2t + 2)u_1(t)$
   (c) $f(t) = \displaystyle\int_0^t (t-\tau)^2 \cos 2\tau d\tau$
   (d) $f(t) = t\cos 3t$
   (e) $f(t) = e^t \delta(t-1)$

7. Find the inverse Laplace transform of the given function.
   (a) $F(s) = \dfrac{2s+6}{s^2 - 4s + 8}$
   (b) $F(s) = \dfrac{e^{-2s}}{s^2 + s - 2}$

8. Solve the initial value problem using the Laplace transform:
   (a) $y'' + 4y = \begin{cases} t, & 0 \le t < 1 \\ 1, & t \ge 1 \end{cases}$, $y(0) = y'(0) = 0$
   (b) $y'' + 2y' + 3y = \delta(t - 3\pi)$, $y(0) = y'(0) = 0$

(c) $y'' + 4y' + 4y = g(t)$, $y(0) = 2$, $y'(0) = -3$

9. Transform the given equation into a system of first order equation, then in matrix notation.

(a) $e^t y'' + t^2 y' - \sin ty = 3 \arctan t, \qquad y(0) = 5, \quad y'(0) = 3.$

(b) $y^{(4)} - \cos ty = 0.$

10. Find $A^{-1}$ if $A = \begin{pmatrix} 1+i & -1+2i \\ 3+2i & 2-i \end{pmatrix}$

11. Find $BA$ if $A = \begin{pmatrix} 1+i & -1+2i \\ 3+2i & 2-i \end{pmatrix}$, $B = \begin{pmatrix} i & 3 \\ 2 & -2i \end{pmatrix}$

12. Find all eigenvalues and eigenvectors of the martix

$$\begin{pmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{pmatrix}$$