MATH 308–518, 519, 520 **Practice Test II** **Spring 2014**

1. The Existence and Uniqueness Theorem guarantees that the solution to

$$x^3y'' + \frac{x}{\sin x}y' - \frac{2}{x-5}\,y = 0, \quad y(2) = 6, \quad y'(2) = 7$$

uniquely exists on

(a) $(-\pi,\ \pi)$
(b) $(0,\ \pi)$
(c) $(5,\ \infty)$
(d) $(0,\ 5)$

2. All of the following pairs of functions form a fundamental set of solutions to some second order differential equation on $(-\infty,\ \infty)$ EXCEPT

(a) $1, \quad e^{-t}$
(b) $\cos t, \quad \sin(t + 2\pi)$
(c) $e^{-2t}\cos 2t, \quad e^{-2t}\sin 2t$
(d) $e^{5t}, \quad e^{5t-1}$

3. Which of the following will be a particular solution to the equation

$$4y'' + 4y' + y = 24x\mathrm{e}^{\frac{x}{2}}?$$

(a) $x^2(Ax + B)\mathrm{e}^{\frac{x}{2}}$
(b) $(Ax + B)\mathrm{e}^{\frac{x}{2}}$
(c) $x(Ax + B)\mathrm{e}^{\frac{x}{2}}$
(d) $(Ax + B)\sin\frac{x}{2} + (Cx + D)\cos\frac{x}{2}$

4. A 2-kg mass is attached to a spring with stiffness $k = 50$ N/m. The damping force is negligible. What is the resonance frequency for the system?

(a) 5
(b) 2
(c) 3
(d) 4

5. The motion of the mass-spring system with damping is governed by $y'' + 2y' + y = 0, \quad y(0) = 1, y'(0) = -3$. This motion is

   (a) undamped
   (b) underdamped
   (c) critically damped
   (d) overdamped

6. $e^{2+\frac{3\pi}{4}i} =$

   (a) π
   (b) $\frac{\sqrt{2}}{2}(1+i)e^2$
   (c) $\frac{\sqrt{2}}{2}(1-i)e^2$
   (d) $\frac{\sqrt{2}}{2}(-1+i)e^2$

7. A 2-kg mass is attached to a spring with stiffness $k = 50$ N/m. The mass is displaced 1/4 m to the left of the equilibrium point and given a velocity of 1 m/sec to the left. The damping force is negligible. The amplitude of this vibration is

   (a) $\frac{\sqrt{41}}{20}$
   (b) 1
   (c) $\frac{\sqrt{20}}{41}$
   (d) $\frac{1}{4}$

8. The FSS to the equation $y'' - 2y' + 5y = 0$ is

   (a) $\{\cos x, \sin x\}$
   (b) $\{e^x \cos 2x, e^x \sin 2x\}$
   (c) $\{e^x, xe^x\}$
   (d) $\{e^x, e^{-x}\}$

9. Given that $y_1(x) = -\frac{1}{2}x^2 + \frac{1}{2}x - \frac{3}{4}$ is a solution to $y'' - y' - 2y = x^2$ and $y_2(x) = \frac{1}{4}e^{3x}$ is a solution to $y'' - y' - 2y = e^{3x}$. A solution to $y'' - y' - 2y = 2x^2 - e^{3x}$ is

   (a) $-x^2 + x - \frac{3}{2} - \frac{1}{4}e^{3x}$
   (b) $x^2 - x - \frac{3}{2} - \frac{1}{4}e^{3x}$
   (c) $-x^2 + x + \frac{3}{4} - e^{3x}$
   (d) $x - \frac{3}{2} - \frac{1}{4}e^{3x}$

10. The Wronskian of two functions $y_1(x) = x + 2x^2$ an $y_2(x) = 2^x$ is

   (a) $2^x(1 + 4x - x(1 + 2x))$
   (b) $-2^x(1 + 4x - x\ln 2(1 + 2x))$
   (c) $(1 + 4x - x(1 + 2x))$
   (d) $2^x(1 + 2x - x\ln 2(1 + 4x))$

11. Find a general solution to the equation

$$y'' + 6y' + 9y = \frac{e^{-3x}}{1 + 2x}$$

12. Find a general solution to the equation

$$4y'' + y' = 4x^3 + 48x^2 + 1$$

13. Given that $y_1(x) = x$ is a solution to

$$x^2y'' + xy' - y = 0,$$

find a second solution of this equation on $(0, +\infty)$.