

1. Find the general solution of the given differential equation.

(a) $y' + 2ty = 2te^{-t^2}$.

(b) $y' = \frac{3x^2 - 1}{3 + 2y}$.

(c) $y' = 2x \sec y$.

(d) $ty' + y = 3t \cos t, \quad t > 0$.

2. Find the solution to the initial value problem

(a) $\frac{dy}{dx} = 4x^3y - y, \quad y(1) = -3$.

(b) $\frac{dy}{dx} + \frac{2y}{t} = \frac{\cos t}{t^2}, \quad y(1) = \frac{1}{2}, \quad t > 0$.

(c) $2\sqrt{x}\frac{dy}{dx} = \cos^2 y, \quad y(4) = \frac{\pi}{4}$.

3. Consider the initial value problem

$$y' + 2y = 5 - t, \quad y(0) = y_0$$

Find the value y_0 for which the solution touches, but does not cross the t -axis.

4. Consider a cascade of 2 tanks, with $V_1 = 100$ gal and $V_2 = 200$ gal the volume of brine of the 2 tanks. Each tank also initially contains 550lb of salt. Pure water flows into the first tank at a rate of 5 gal/min. The mixture flows from the first tank to the second one and flows out of the second tank at the same rate (5 gal/min).

Find the amount of salt in the 2 tanks at any time t .

5. A ball with mass 1kg is thrown upward with initial velocity 20 m/s from the roof of a building 50 m high. A force due to the resistance of the air of $v/10$, where the velocity is measured in m/s, acts on the ball.

(a) Find the maximum height above the ground that the ball reaches

(b) Find the time that the ball hits the ground.

6. College graduate borrows \$10,000 to buy a car. The lender charges interest at an annual rate of 10%. Assuming that the interest is compounded continuously and that the borrower makes payment continuously at a constant annual rate k , determine the payment rate k that is required to pay off the loan in 5 years. Also determine how much interest is paid during the 5-year period.

7. Food, initially at a temperature of 40°F, was placed in an oven preheated to 350°F. After 10 minutes in the oven, the food had warmed to 120°F. After 20 minutes, the food was removed from the oven and allowed to cool at room temperature (72°F). If the ideal serving temperature is 110°F, when should the food be served?