1. College graduate borrows \$10,000 to buy a car. The lender charges interest at an annual rate of 10%. Assuming that the interest is compounded continuously and that the borrows makes payment continuously at a constant annual rate k, determine the payment rate $k$ that is required to pay off the loan in 5 years. Also determine how much interest is paid during the 5-year period.

2. Food, initially at a temperature of 40°F, was placed in an oven preheated to 350°F. After 10 minutes in the oven, the food had warmed to 120°F. After 20 minutes, the food was removed from the oven and allowed to cool at room temperature (72°F). If the ideal serving temperature is 110°F, when should the food be served?

3. Determine an interval in which the solutions of the following initial value problems are certain to exist.

   (a) $y' + \dfrac{\sin t}{t^2-1}y = \dfrac{\cot t}{t^2-4t+3}, \qquad y(2) = -1.$

   (b) $t(t-4)y' + t^2\ln(t+5)y = 0, \qquad y(-3) = 7.$

4. State where in the $ty$-plane the hypothesis of theorem 2.4.2 are satisfied.

   (a) $y' = \dfrac{\ln(ty)}{1-(t^2+y^2)}.$

   (b) $y' = (t^2-y)^{1/3}.$

5. Solve the following initial value problems and determine how the interval in which the solution exists depends on the initial value $t_0$.

   (a) $y' = \dfrac{-4}{t}y, \qquad y(t_0) = y_0$

   (b) $y' + y^3 = 0 \qquad y(t_0) = y_0.$

6. Verify that both $y_1 = 1-t$ and $y_2 = \dfrac{-t^2}{4}$ are solutions to the same initial value problem

$$y'(t) = \frac{-t + (t^2+4y)^{(1/2)}}{2}, \qquad y(2) = -1.$$

   Does it contradict the existence and uniqueness theorem?

7. Given the differential equation

$$y'(t) = y^3 - 2y^2 + y$$

   (a) Find the equilibrium solutions

   (b) Graph the phase line. Classify each equilibrium solution as either stable, unstable, semistable.

   (c) Determine where the solutions are concave up? concave down?

   (d) Sketch the graph of the solutions

8. Given the differential equation $y' = y(y-2)$

   (a) Sketch the graph of $f$ versus $y$.

   (b) Find the equilibrium solutions

   (c) Graph the phase line. Classify each equilibrium solution as either stable, unstable, semistable.

   (d) Determine where the solutions are concave up? concave down?

   (e) Sketch the graph of the solutions