1. Express $f(t)$ in terms of the unit step function $u_c(t)$ and find its Laplace transform.

   (a) $f(t) = \begin{cases} t^2, & 0 \le t < 2 \\ e^t, & 2 \le t \end{cases}$.

(b) $f(t) = \begin{cases} 2, & 0 \leq t < 3 \\ 5t^2, & 3 \leq t < 8 \\ 3\cos(t-8), & 8 \leq t \end{cases}$.

2. Find the inverse Laplace transform of the given functions

   (a) $F(s) = \dfrac{s + 3se^{-5s}}{s^2 - 4s + 3}$

(b) $F(s) = \dfrac{(2s-1)e^{-s}}{s^2 - 2s + 2}$.

3. Find the solution to the given initial value problem

(a) $y'' + 3y' + 2y = \begin{cases} 1, & 0 \le t < 10 \\ 0, & 10 \le t \end{cases}, \qquad y(0) = 0, \quad y'(0) = 0.$

(b) $y'' + 2y' + 5y = \sin(t) + u_\pi(t)\cos(t - \pi), \qquad y(0) = 0, \quad y'(0) = 0.$

(c) $y'' + 2y' + 2y = \cos t + \delta(t - \pi/2), \qquad y(0) = 0, \quad y'(0) = 0.$

(d) $y'' - y' - 6y = g(t), \qquad y(0) = 1, \quad y'(0) = 8.$

4. Find the inverse Laplace transform of the given function by using the convolution theorem.

(a) $F(s) = \dfrac{(3s-3)e^{-5s}}{s^2+2s+10}$.

(b) $F(s) = \dfrac{1}{s^4(s^2-1)}$.

(c) $F(s) = \dfrac{s}{(s+1)^2(s+4)^3}$.

5. Find the Laplace transform of

(a) $f(t) = \int_0^1 (t-\tau)e^{3\tau}\,d\tau$

(b) $f(t) = \int_0^1 e^{\tau} \sin(t - \tau) \; d\tau$

6. Transform the given equation into a system of first order equation, then in matrix notation.

   (a) $e^t y'' + t^2 y' - \sin t y = 3 \arctan t, \qquad y(0) = 5, \quad y'(0) = 3.$

(b) $y^{(4)} - \cos t y = 0$.

7. If $A = \begin{pmatrix} 1 & -2 & 0 \\ 3 & 2 & -1 \\ -2 & 1 & 2 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & -2 & 3 \\ -1 & 5 & 0 \\ 6 & 1 & 2 \end{pmatrix}$, find

(a) $3A - 2B$

(b) $AB - BA$

8. Verify that the vector $X(t) = \begin{pmatrix} 6 \\ -8 \\ -4 \end{pmatrix} e^{-t} + 2 \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} e^{2t}$ is solution to the system

$$X' = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}$$

9. Verify that $\psi = \begin{pmatrix} e^{-3t} & e^{2t} \\ -4e^{-3t} & e^{2t} \end{pmatrix}$ is solution to

$$\psi' = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix} \psi$$