

1. Transform the given equation into a system of first order equation, then in matrix notation.

(a)  $e^t y'' + t^2 y' - \sin ty = 3 \arctan t, \quad y(0) = 5, \quad y'(0) = 3.$

(b)  $y'' - \cos ty' + 3ty = 0.$

2. Transform the given system into a single equation of second order. Find  $x_1$  and  $x_2$  that satisfies the initial conditions when initial conditions are given:

(a) 
$$\begin{cases} x_1' = x_1 - 2x_2, & x_1(0) = -1 \\ x_2' = 3x_1 - 4x_2, & x_2(0) = 2 \end{cases}.$$

(b) 
$$\begin{cases} x_1' + 2x_2' = 4x_1 + 5x_2 \\ 2x_1' - x_2' = 3x_1 \end{cases}$$

3. Verify that  $\psi = \begin{pmatrix} e^{-3t} & e^{2t} \\ -4e^{-3t} & e^{2t} \end{pmatrix}$  is solution to

$$\psi' = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix} \psi$$

4. Given the matrices

$$A = \begin{pmatrix} -2 & 1 & 2 \\ 1 & 0 & -3 \\ 2 & -1 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 2 & 3 \\ 3 & -1 & -1 \\ -2 & 1 & 0 \end{pmatrix}$$

(a) Find  $3A - 2B$

(b) Calculate  $\det(A)$ .

(c) Find  $A^2 - AB$ .

5. Given the matrices and vectors

$$A = \begin{pmatrix} -2 & 0 & 1 \\ 1 & -1 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 3 \\ -1 & 0 \\ 3 & -1 \end{pmatrix}, \quad X(t) = \begin{pmatrix} 2t \\ e^{-t} \end{pmatrix}, \quad Y(t) = \begin{pmatrix} \cos t \\ \sin t \\ t \end{pmatrix}.$$

(a) Find  $AB$  and  $BA$

(b) Find  $AX, BX, AY, BY$  if possible.

6. Determine whether the vectors are linearly independent. If they are linearly dependent, find a linear relation among them:

(a)  $X_1 = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \quad X_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad X_3 = \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}.$

(b)  $X_1 = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}, \quad X_2 = \begin{pmatrix} 5 \\ 1 \\ 0 \end{pmatrix}.$

7. Are the vector functions linearly independent? If they are linearly dependent, find a linear relation among them.

$$(a) X_1(t) = \begin{pmatrix} e^{-3t} \\ -4e^{-3t} \end{pmatrix} \quad X_2(t) = \begin{pmatrix} e^{-3t} \\ e^{-3t} \end{pmatrix}.$$

$$(b) X_1(t) = \begin{pmatrix} 2e^t \\ -e^t \end{pmatrix}, \quad X_2(t) = \begin{pmatrix} e^t \\ 2e^t \end{pmatrix} \quad X_3(t) = \begin{pmatrix} 3te^t \\ te^t \end{pmatrix}.$$

8. Find the eigenvalues and eigenvectors of the given matrix

$$(a) A = \begin{pmatrix} -2 & 1 \\ -1 & -2 \end{pmatrix}$$

$$(b) A = \begin{pmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{pmatrix}.$$

$$(c) A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & -2 & 1 \\ 2 & 1 & -2 \end{pmatrix}.$$