1. Transform the given equation into a system of first order equation, then in matrix notation.

   (a) $e^t y'' + t^2 y' - \sin t y = 3 \arctan t, \qquad y(0) = 5, \quad y'(0) = 3.$

   (b) $y'' - \cos t y' + 3ty = 0.$

2. Transform the given system into a single equation of second order. Find $x_1$ and $x_2$ that satisfies the initial conditions when initial conditions are given:

(a) $$\begin{cases} x_1' = x_1 - 2x_2, & x_1(0) = -1 \\ x_2' = 3x_1 - 4x_2, & x_2(0) = 2 \end{cases}.$$

(b) $\begin{cases} x_1' + 2x_2' = 4x_1 + 5x_2 \\ 2x_1' - x_2' = 3x_1 \end{cases}$

3. Verify that $\psi = \begin{pmatrix} e^{-3t} & e^{2t} \\ -4e^{-3t} & e^{2t} \end{pmatrix}$ is solution to

$$\psi' = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix} \psi$$

4. Given the matrices

$$A = \begin{pmatrix} -2 & 1 & 2 \\ 1 & 0 & -3 \\ 2 & -1 & 1 \end{pmatrix} \qquad B = \begin{pmatrix} 1 & 2 & 3 \\ 3 & -1 & -1 \\ -2 & 1 & 0 \end{pmatrix}$$

(a) Find $3A - 2B$

(b) Calculate $\det(A)$.

(c) Find $A^2 - AB$.

5. Given the matrices and vectors

$$A = \begin{pmatrix} -2 & 0 & 1 \\ 1 & -1 & 3 \end{pmatrix}, \qquad B = \begin{pmatrix} 1 & 3 \\ -1 & 0 \\ 3 & -1 \end{pmatrix}, \qquad X(t) = \begin{pmatrix} 2t \\ e^{-t} \end{pmatrix}, \qquad Y(t) = \begin{pmatrix} \cos t \\ \sin t \\ t \end{pmatrix}.$$

(a) Find $AB$ and $BA$

(b) Find $AX$, $BX$, $AY$, $BY$ if possible.

6. Determine whether the vectors are linearly independent. If they are linearly dependent, find a linear relation among them:

(a) $X_1 = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}, \qquad X_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \qquad X_3 = \begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}.$

(b) $X_1 = \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}, \qquad X_2 = \begin{pmatrix} 5 \\ 1 \\ 0 \end{pmatrix}.$

7. Are the vector functions linearly independent? If they are linearly dependent, find a linear relation among them.

(a) $X_1(t) = \begin{pmatrix} e^{-3t} \\ -4e^{-3t} \end{pmatrix} \qquad X_2(t) = \begin{pmatrix} e^{-3t} \\ e^{-3t} \end{pmatrix}.$

(b) $X_1(t) = \begin{pmatrix} 2e^{t} \\ -e^{t} \end{pmatrix}, \qquad X_2(t) = \begin{pmatrix} e^{t} \\ 2e^{t} \end{pmatrix} \qquad X_3(t) = \begin{pmatrix} 3te^{t} \\ te^{t} \end{pmatrix}.$

8. Find the eigenvalues and eigenvectors of the given matrix

(a) $A = \begin{pmatrix} -2 & 1 \\ -1 & -2 \end{pmatrix}$

(b) $A = \begin{pmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{pmatrix}$.

(c) $A = \begin{pmatrix} 1 & 0 & 0 \\ 2 & -2 & 1 \\ 2 & 1 & -2 \end{pmatrix}$.