1. A large tank initially contains 10 L of fresh water. A brine containing 20 g/L of salt flows into the tank at a rate of 3 L/min. The solution inside the tank is kept well stirred and flows out of the tank at the rate 2 L/min. Determine the concentration of salt in the tank as a function of time.

2. An object with temperature $150°$ is placed in a freezer whose temperature is $30°$. Assume that the temperature of the freezer remains essentially constant.

   (a) If the object is cooled to $120°$ after 8 min, what will its temperature be after 18 min?

   (b) When will its temperature be $60°$?

3. Determine (without solving the problem) an interval in which the solution to the initial value problem

$$(4 - t^2)y' + 2ty = 3t^2, \qquad y(1) = -3$$

   is certain to exist.

4. Solve the initial value problem

$$y' = \frac{t^2}{1 + t^3}, \qquad y(0) = y_0$$

   and determine how the interval in which the solution exists depends on the initial value y_0.

5. Solve the following initial value problem

$$\sqrt{y}dt + (1 + t)dy = 0 \quad y(0) = 1.$$

6. Find the general solution to the equation

$$(t^2 - 1)y' + 2ty + 3 = 0$$

7. For the equation $\dfrac{dy}{dt} = y^3 - 2y^2 + y$

   (a) find the equilibrium solutions

   (b) sketch the phase line and determine whether the equilibrium solutions are stable, unstable, or semistable

   (c) graph some solutions

   (d) if $y(t)$ is the solution of the equation satisfying the initial condition $y(0) = y_0$, where $-\infty \le y_0 \le \infty$, find the limit of $y(t)$ when $t \to \infty$ and the limit of $y(t)$ when $t \to -\infty$

   (e) solve the equation.

8. Solve the initial value problem

$$(ye^{xy} \cos(2x) - 2e^{xy} \sin(2x) + 2x)dx + (xe^{xy} \cos(2x) - 3)dy = 0, \qquad y(0) = -1$$

9. Find an integrating factor for the equation

$$(3xy + y^2) + (x^2 + xy)y' = 0$$

   and then solve the equation.

10. Solve the equation/initial value problem

   (a) $6y'' - 5y' + y = 0$, $y(0) = 4$, $y'(0) = 0$

(b) $4y'' - 12y' + 9y = 0$

(c) $y'' + 4y' + 5y = 0$, $y(0) = 0$, $y'(0) = 1$

11. Determine the longest interval in which the given initial value problem is certain to have a unique solution.

$$(x - 2)y'' + y' + (x - 2)(\tan x)y = 0, \qquad y(3) = 1, y'(3) = 2.$$

12. If the Wronskian of f and g is $3e^{4t}$ and $f(t) = e^{2t}$, find $g(t)$.

13. A spring is stretch 10 cm by a force of 3 N. A mass of 2 kg is hung from the spring and is also attached to a viscous damper that exerts a force of 3 N when the velocity of the mass 5 m/s. If the mass is pulled down 5 cm below its equilibrium position and given an initial velocity of 10 cm/s, determine its position u at any time. Find the quasifrequency of the motion.

14. A mass weighting 8 lb is attached to a spring hanging from the ceiling and comes to rest at its equilibrium position. At $t = 0$, an external force $F(t) = 2\cos 2t$ lb is applied to the system. If the spring constant is 10 lb/ft and the damping constant is 1 lb-sec/ft, find the steady-state solution for the system.

15. A mass weighing 4 lb stretches a spring 1.5 in. The mass is given a positive displacement 2 in from its equilibrium position and released with no initial velocity. Assuming that there is no damping and the mass is acted on by an external force of $2\cos 3t$ lb,

(a) Formulate the initial value problem describing the motion of mass

(b) Solve the initial value problem.

(c) If the given external force is replaced by a force $4\cos\omega t$ of frequency ω, find the value of ω for which resonance occurs.

16. Find the general solution of the equation

(a) $y'' + 6y' + 9y = \frac{e^{-3x}}{1+2x}$

(b) $y'' + 2y' + y = 4e^{-t}$, $y(0) = 2$, $y'(0) = 1$

(c) $y'' + 4y = 32\sin 2t - 32t\cos 2t$

17. For the equation $y'' + xy' + 2y = 0$

(a) Seek its power series solution about $x_0 = 0$; find the recurrence relation.

(b) Find the first four terms in each of two solutions y_1 and y_2. Show that $W[y_1, y_2](0) \neq 0$.

18. Determine a lower bound for the radius of convergence of series solution for the equation

$$(x^2 + x)y'' + 3y' - 6xy = 0$$

about

(a) $x_0 = -2$

(b) $x_0 = -\dfrac{1}{3}$

(c) $x_0 = 2$

19. Determine $y'''(1)$ if $y(x)$ is the solution of the initial value problem

$$x^2 y'' + (1 + x)y' + 3(\ln x)y = 0, \qquad y(1) = 2, y'(1) = 0$$

20. Find the Laplace transform of the given function.

(a) $f(t) = \begin{cases} \dfrac{t}{2}, & 0 \le t < 6 \\ 3, & t \ge 6 \end{cases}$

(b) $f(t) = (t^2 - 2t + 2)u_1(t)$

(c) $f(t) = \displaystyle\int_0^t (t - \tau)^2 \cos 2\tau \, d\tau$

(d) $f(t) = t \cos 3t$

(e) $f(t) = e^t \delta(t - 1)$

21. Find the inverse Laplace transform of the given function.

(a) $F(s) = \dfrac{2s + 6}{s^2 - 4s + 8}$

(b) $F(s) = \dfrac{e^{-2s}}{s^2 + s - 2}$

22. Solve the initial value problem using the Laplace transform:

(a) $y'' + 4y = \begin{cases} t, & 0 \le t < 1 \\ 1, & t \ge 1 \end{cases}$, $y(0) = y'(0) = 0$

(b) $y'' + 2y' + 3y = \delta(t - 3\pi)$, $y(0) = y'(0) = 0$

(c) $y'' + 4y' + 4y = g(t)$, $y(0) = 2$, $y'(0) = -3$

23. Find A^{-1} if $A = \begin{pmatrix} 1 + i & -1 + 2i \\ 3 + 2i & 2 - i \end{pmatrix}$

24. Find BA if $A = \begin{pmatrix} 1 + i & -1 + 2i \\ 3 + 2i & 2 - i \end{pmatrix}$, $B = \begin{pmatrix} i & 3 \\ 2 & -2i \end{pmatrix}$

25. Find the general solution of the system. Classify the critical point $(0,0)$ as to type, determine whether it is stable or unstable, sketch the phase portrait.

(a) $\mathbf{x}' = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix} \mathbf{x}$

(b) $\mathbf{x}' = \begin{pmatrix} -3 & -1 \\ 1 & -1 \end{pmatrix} \mathbf{x}$

(c) $\mathbf{x}' = \begin{pmatrix} -3 & 2 \\ -1 & -1 \end{pmatrix} \mathbf{x}$

26. Find the general solution of the system using variation of parameters and Laplace Transform, if possible.

(a) $\mathbf{x}' = \begin{pmatrix} 1 & 1 \\ 4 & -2 \end{pmatrix} \mathbf{x} + \begin{pmatrix} e^{-2t} \\ -2e^t \end{pmatrix}$

(b) $\mathbf{x}' = \begin{pmatrix} 4 & -2 \\ 8 & -4 \end{pmatrix} \mathbf{x} + \begin{pmatrix} t^{-3} \\ -t^{-2} \end{pmatrix}$