

1. Find the general solution of the given differential equation.

(a) $y' + 2ty = 2te^{-t^2}$.

(b) $2\sqrt{x}y' = \sqrt{1-y^2}$.

(c) $y' = 2x \sec y$.

(d) $ty' + y = 3t \cos t, \quad t > 0.$

2. Find the solution to the initial value problem

(a) $\frac{dy}{dx} = 4x^3y - y, \quad y(1) = -3.$

(b) $2\sqrt{x}\frac{dy}{dx} = \cos^2 y, \quad y(4) = \frac{\pi}{4}.$

$$(c) \frac{dy}{dx} + \frac{2y}{t} = \frac{\cos t}{t^2} \quad y(1) = \frac{1}{2}, \quad t > 0.$$

3. Consider the initial value problem

$$y' + 2y = 5 - t, \quad y(0) = y_0$$

Find the value y_0 for which the solution touches, but does not cross the t -axis.

4. A 120-gallon tank initially contains 90 lb of salt dissolved in 90 gallons of water. Brine containing 2 lb/gal of salt flows into the tank at the rate of 4 gal/min, and the well-stirred mixture flows out of the tank at a rate of 3 gal/min. How much salt does the tank contain when it is full?

5. In a certain culture of bacteria, the number of bacteria increases sixfold in 10 hrs. How long does it take for the population to double?

6. A cake is removed from an oven at 210°F and left to cool at room temperature, which is 70°F . After 30 min the temperature of the cake is 140°F . When will it be 100°F ?

7. A ball with mass 1kg is thrown upward with initial velocity 20 m/s from the roof of a building 50 m high. A force due to the resistance of the air of $v/10$, where the velocity is measured in m/s, acts on the ball. Find the maximum height above the ground that the ball reaches.