1. Determine an interval in which the solutions of the following initial value problems are certain to exist.

   (a) $y' + \dfrac{\sin t}{t^2-1}y = \dfrac{\cot t}{t^2-4t+3}, \qquad y(2) = -1.$

   (b) $t(t-4)y' + t^2\ln(t+5)y = 0, \qquad y(-3) = 7.$

2. State where in the ty-plane the hypothesis of theorem 2.4.2 are satisfied.

   (a) $y' = \dfrac{\ln(ty)}{1-(t^2+y^2)}.$

   (b) $y' = (t^2-y)^{1/3}.$

3. Solve the following initial value problems and determine how the interval in which the solution exists depends on the initial value t_0.

   (a) $y' = \dfrac{-4}{t}y, \qquad y(t_0) = y_0$

   (b) $y' + y^3 = 0 \qquad y(t_0) = y_0.$

4. Verify that both $y_1 = 1-t$ and $y_2 = \dfrac{-t^2}{4}$ are solutions to the same initial value problem

$$y'(t) = \frac{-t+(t^2+4y)^{(1/2)}}{2}, \qquad y(2) = -1.$$

   Does it contradict the existence and uniqueness theorem?

5. Given the differential equation $y' = y(y-2)$

   (a) Find the equilibrium solutions

   (b) Graph the phase line. Classify each equilibrium solution as either stable, unstable, semistable.

   (c) Graph some solutions

   (d) If $y(t)$ is the solution of the equation satisfying the initial condition $y(0) = y_0$, where $-\infty < y_0 < \infty$, find the limit of $y(t)$ when $t \to \infty$

6. Given the differential equation

$$y'(t) = y^3 - 2y^2 + y$$

   (a) Find the equilibrium solutions

   (b) Graph the phase line. Classify each equilibrium solution as either stable, unstable, semistable.

   (c) Graph some solutions

   (d) If $y(t)$ is the solution of the equation satisfying the initial condition $y(0) = y_0$, where $-\infty < y_0 < \infty$, find the limit of $y(t)$ when $t \to \infty$

7. Determine if the equations are exact and solve the ones that are:

   (a) $(2x+5y)dx + (5x-6y)dy = 0.$

   (b) $1 + \dfrac{y}{x} - \frac{1}{x}y' = 0.$

   (c) $(\sin(2t)+2y)dy + (2y\cos(2t)-6t^2)dt = 0.$

8. Show that the equations are not exact. However, if you multiply by the given integrating factor, you can solve the resulting exact equations.

   (a) $(x^2+y^2-x)dx - ydy = 0 \qquad \mu(x,y) = \frac{1}{x^2+y^2}$

   (b) $3(y+1)dx - 2xdy = 0, \qquad \mu(x,y) = \frac{y+1}{x^4}$

9. Find an integrating factor for the equation

$$(3xy+y^2) + (x^2+xy)y' = 0$$

   and then solve the equation.