1. Determine an interval in which the solutions of the following initial value problems are certain to exist.

   (a) $y' + \dfrac{\sin t}{t^2 - 1} y = \dfrac{\cot t}{t^2 - 4t + 3}, \qquad y(2) = -1.$

   (b) $t(t - 4)y' + t^2 \ln(t + 5)y = 0, \qquad y(-3) = 7.$

2. State where in the ty-plane the hypothesis of theorem 2.4.2 are satisfied.

(a) $y' = \dfrac{\ln(ty)}{1 - (t^2 + y^2)}.$

(b) $y' = (t^2 - y)^{1/3}.$

3. Solve the following initial value problems and determine how the interval in which the solution exists depends on the initial value t_0.

(a) $y' = \dfrac{-4}{t} y, \qquad y(t_0) = y_0$

(a) $y' = \dfrac{-4}{t} y, \qquad y(t_0) = y_0$

(b) $y' + y^3 = 0$ $\qquad y(t_0) = y_0$.

4. Verify that both $y_1 = 1 - t$ and $y_2 = \dfrac{-t^2}{4}$ are solutions to the same initial value problem

$$y'(t) = \frac{-t + (t^2 + 4y)^{(1/2)}}{2}, \qquad y(2) = -1.$$

Does it contradict the existence and uniqueness theorem?

5. Given the differential equation $y' = y(y - 2)$

   (a) Find the equilibrium solutions

   (b) Graph the phase line. Classify each equilibrium solution as either stable, unstable, semistable.

   (c) Graph some solutions

   (d) If $y(t)$ is the solution of the equation satisfying the initial condition $y(0) = y_0$, where $-\infty < y_0 < \infty$, find the limit of $y(t)$ when $t \to \infty$

6. Given the differential equation
$$y'(t) = y^3 - 2y^2 + y$$

(a) Find the equilibrium solutions

(b) Graph the phase line. Classify each equilibrium solution as either stable, unstable, semistable.

(c) Graph some solutions

(d) If $y(t)$ is the solution of the equation satisfying the initial condition $y(0) = y_0$, where $-\infty < y_0 < \infty$, find the limit of $y(t)$ when $t \to \infty$

7. Determine if the equations are exact and solve the ones that are:

   (a) $(2x + 5y)dx + (5x - 6y)dy = 0$.

(b) $1 + \dfrac{y}{x} - \dfrac{1}{x}y' = 0.$

(c) $(\sin(2t) + 2y)dy + (2y\cos(2t) - 6t^2)dt = 0.$

8. Show that the equations are not exact. However, if you multiply by the given integrating factor, you can solve the resulting exact equations.

(a) $(x^2 + y^2 - x)dx - ydy = 0$ $\mu(x, y) = \frac{1}{x^2 + y^2}$

(b) $3(y + 1)dx - 2xdy = 0,$ $\mu(x, y) = \frac{y+1}{x^4}$

9. Find an integrating factor for the equation

$$(3xy + y^2) + (x^2 + xy)y' = 0$$

and then solve the equation.