

1. Use the definition to find the Laplace transforms of

(a)  $f(t) = e^{at}$  where  $a$  is a non zero real number.

(b)  $f(t) = \begin{cases} 5-t & 0 \leq t < 2 \\ 3t & 2 \leq t. \end{cases}$

2. Find the inverse Laplace transform of

(a)  $F(s) = \frac{4}{(s-2)^5}$ .

(b)  $F(s) = \frac{8s^2 - 4s + 12}{s(s^2 + 4)}$ .

(c)  $F(s) = \frac{2s - 3}{s^2 + 2s + 10}$ .

3. Use the Laplace transform to solve the given initial value problem

(a)  $y'' + 3y' + 2y = 4t$ ,  $y(0) = 1$ ,  $y'(0) = 0$ .

(b)  $y'' + 9y = \cos 2t$ ,  $y(0) = 0$ ,  $y'(0) = 1$ .

4. Express  $f(t)$  in terms of the unit step function  $u_c(t)$  and find its Laplace transform.

(a)  $f(t) = \begin{cases} t^2, & 0 \leq t < 2 \\ e^t, & 2 \leq t \end{cases}$ .

(b)  $f(t) = \begin{cases} 2, & 0 \leq t < 3 \\ 5t^2, & 3 \leq t < 8 \\ 3 \cos(t-8), & 8 \leq t \end{cases}$ .

5. Find the inverse Laplace transform of the given functions

(a)  $F(s) = \frac{s + 3se^{-5s}}{s^2 - 4s + 3}$

(b)  $F(s) = \frac{(2s-1)e^{-s}}{s^2 - 2s + 2}$ .

6. Find the solution to the given initial value problem

(a)  $y'' + 3y' + 2y = \begin{cases} 1, & 0 \leq t < 10 \\ 0, & 10 \leq t \end{cases}$ ,  $y(0) = 0$ ,  $y'(0) = 0$ .

(b)  $y'' + 4y = u_\pi(t) - u_{3\pi}(t)$ ,  $y(0) = 1$ ,  $y'(0) = -2$

(c)  $y'' + 2y' + 5y = \sin(t) + u_\pi(t) \cos(t - \pi)$ ,  $y(0) = 0$ ,  $y'(0) = 0$ .