

1. Find the general solution of the equation  $y'' + 6y' + 9y = \frac{e^{-3x}}{1 + 2x}$
2. A mass weighing 3 lb stretches a spring 3 in. If the mass is pushed upward, contracting the spring a distance of 1 in. then set in motion with a downward velocity of 2 ft/s, and if there is no damping, find the position  $u$  of the mass at any time  $t$ . Determine the frequency, period and amplitude of the motion.
3. A spring is stretch 10 cm by a force of 3 N. A mass of 2 kg is hung from the spring and is also attached to a viscous damper that exerts a force of 3 N when the velocity of the mass 5 m/s. If the mass is pulled down 5 cm below its equilibrium position and given an initial velocity of 10 cm/s, determine its position  $u$  at any time. Find the quasifrequency of the motion.
4. A mass weighing 4 lb stretches a spring 1.5 in. The mass is given a positive displacement 2 in from its equilibrium position and released with no initial velocity. Assuming that there is no damping and the mass is acted on by an external force of  $2 \cos 3t$  lb,
  - (a) Formulate the initial value problem describing the motion of mass
  - (b) Solve the initial value problem.
  - (c) If the given external force is replaced by a force  $4 \cos \omega t$  of frequency  $\omega$ , find the value of  $\omega$  for which resonance occurs.
5. A spring is stretched 6 in by a mass that weighs 8 lb. The mass is attached to a dashpot mechanism that has a damping constant of 0.25 lb·s/ft and is acted by an external force of  $4 \cos 2t$  lb.
  - (a) Find the steady-state response of this system.
  - (b) if the given mass is replaced by a mass  $m$ , determine the value of  $m$  for which the amplitude of the steady-state response is maximum.
6. Find the inverse Laplace transform of the given function by using the convolution theorem.
  - (a)  $F(s) = \frac{1}{s^4(s^2 - 1)}$ .
  - (b)  $F(s) = \frac{s}{(s + 1)^2(s + 4)^3}$ .
7. Find the Laplace transform of
  - (a)  $f(t) = \int_0^t (t - \tau)e^{3\tau} d\tau$
  - (b)  $f(t) = \int_0^t e^\tau \sin(t - \tau) d\tau$
8. Find the solution of the initial value problem
  - (a)  $y'' + 3y' + 2y = \begin{cases} 1, & 0 \leq t < 10 \\ 0, & 10 \leq t \end{cases}$ ,  $y(0) = 0$ ,  $y'(0) = 0$ .
  - (b)  $y'' + 2y' + 2y = \cos t + \delta(t - \pi/2)$ ,  $y(0) = 0$ ,  $y'(0) = 0$ .
  - (c)  $y'' - y' - 6y = g(t)$ ,  $y(0) = 1$ ,  $y'(0) = 8$ .
9. For the equation  $y'' + xy' + 2y = 0$ 
  - (a) Seek its power series solution about  $x_0 = 0$ ; find the recurrence relation.
  - (b) Find the general term of each solution  $y_1(x)$  and  $y_2(x)$ .
  - (c) Find the first four terms in each of two solutions  $y_1$  and  $y_2$ . Show that  $W[y_1, y_2](0) \neq 0$ .