Name_____ ID_____

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MATH 251 Quiz 1 Fall 2006

Sections 507 Solutions P. Yasskin

Multiple Choice & Work Out: (5 points each)

1. Find the equation of a sphere if one of its diameters has endpoints $(1,0,3)$ and $(7,8,-21)$.

a. $(x+4)^2+(y+4)^2+(z-9)^2=169$
b. $(x+4)^2+(y+4)^2+(z-9)^2=13$
c. $(x-4)^2+(y-4)^2+(z+9)^2=169$ Correct Choice
d. $(x-4)^2+(y-4)^2+(z+9)^2=13$
e. $(x-4)^2+(y+4)^2+(z+9)^2=13$

The center is the midpoint: $(p,q,r)=\dfrac{(1,0,3)+(7,8,-21)}{2}=(4,4,-9)$

The radius is the distance from the center to one endpoint: $R=\sqrt{3^2+4^2+12^2}=13$

The circle is: $(x-4)^2+(y-4)^2+(z+9)^2=169$

2. If $\vec{u}$ points North and $\vec{v}$ points SouthEast, then $\vec{u}\times\vec{v}$ points

a. Up (away from the center of the earth)
b. Down (toward the center of the earth) Correct Choice
c. SouthWest
d. WestSouthWest
e. EastNorthEast

Put your fingers North with the palm facing SouthEast, your thumb points Down.

3. Find the equation of the plane through the points $P=(2,1,2)$, $Q=(3,4,2)$ and $R=(2,2,5)$. What is the z-intercept?.

a. 17 Correct Choice
b. 20
c. 23
d. 26
e. 27

$\overrightarrow{PQ}=Q-P=\langle 1,3,0\rangle$ $\overrightarrow{PR}=R-P=\langle 0,1,3\rangle$ $\vec{N}=\overrightarrow{PQ}\times\overrightarrow{PR}=\begin{vmatrix}\hat{\imath}&\hat{\jmath}&\hat{k}\\1&3&0\\0&1&3\end{vmatrix}=\langle 9,-3,1\rangle$

$\vec{N}\cdot X=\vec{N}\cdot P$ $9x-3y+z=9\cdot 2-3\cdot 1+1\cdot 2=17$ $z=-9x+3y+17$ z-intercept $=17$

4. For what value of x is the scalar projection of $\vec{b} = \langle 2, 2x, x+1 \rangle$ onto $\vec{a} = \langle 4, 3, 0 \rangle$ equal to 1?

a. $x = -2$

b. $x = -\frac{3}{2}$

c. $x = -1$

d. $x = -\frac{1}{2}$ Correct Choice

e. $x = \frac{1}{2}$

$\text{comp}_{\vec{a}}\vec{b} = \frac{\vec{b} \cdot \vec{a}}{|\vec{a}|} = \frac{8+6x}{5} = 1 \qquad 8+6x = 5 \qquad 6x = -3 \qquad x = -\frac{1}{2}$

5. Find the point where the line $(x, y, z) = (1-t, -3+2t, 1-2t)$ intersects the plane $(x, y, z) = (2-r-s, 1+2r, 3)$ or show they don't intersect.

Equate the line and the plane:

$$\begin{aligned} 1 - t &= 2 - r - s \\ -3 + 2t &= 1 + 2r \\ 1 - 2t &= 3 \end{aligned}$$

Solve for r, s and t:

$$\begin{aligned} r + s - t &= 1 \\ -2r + 2t &= 4 \\ -2t &= 2 \end{aligned} \quad \Rightarrow \quad \begin{aligned} t &= -1 \\ r &= -3 \\ s &= 3 \end{aligned}$$

Plug back into the line: $(x, y, z) = (1-(-1), -3+2(-1), 1-2(-1)) = (2, -5, 3)$

Check:

Plug back into the plane: $(x, y, z) = (2-(-3)-(3), 1+2(-3), 3) = (2, -5, 3)$