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MATH 251	Quiz 3	Fall 2006
Sections 507	Solutions	P. Yasskin

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Multiple Choice: (5 points each)

1. For the function $f(x,y) = y^2\cos(xy)$ which partial derivative is incorrect?

a. $\dfrac{\partial f}{\partial x} = -y^3\sin(xy)$

b. $\dfrac{\partial f}{\partial y} = 2y\cos(xy) - xy^2\sin(xy)$

c. $\dfrac{\partial^2 f}{\partial x^2} = -y^4\cos(xy)$

d. $\dfrac{\partial^2 f}{\partial y\partial x} = -3y^2\sin(xy) - xy^3\cos(xy)$

e. $\dfrac{\partial^2 f}{\partial x\partial y} = -y^2\sin(xy) - xy^3\cos(xy)$ Correct Choice

Use product rule and chain rule:

$$\frac{\partial^2 f}{\partial x\partial y} = \frac{\partial}{\partial x}(2y\cos(xy) - xy^2\sin(xy)) = -2y^2\sin(xy) - y^2\sin(xy) - xy^3\cos(xy)$$

2. Find the equation of the plane tangent to $z = x^2y^3$ at the point $(2,1,4)$.
Its z-intercept is:

a. 0

b. -24

c. -16 Correct Choice

d. 24

e. 4

$f(x,y) = x^2y^3 \qquad f(2,1) = 4$

$\dfrac{\partial f}{\partial x} = 2xy^3 \qquad \dfrac{\partial f}{\partial x}(2,1) = 4$

$\dfrac{\partial f}{\partial y} = 3x^2y^2 \qquad \dfrac{\partial f}{\partial y}(2,1) = 12$

$$z = f_{\tan}(x,y) = f(2,1) + \frac{\partial f}{\partial x}(2,1)(x-2) + \frac{\partial f}{\partial y}(2,1)(y-1) = 4 + 4(x-2) + 12(y-1)$$

$z = 4x + 12y - 16$ The z-intercept is -16

3. Consider a function $p(x,y)$. If $p(2,3) = 3$, $\frac{\partial p}{\partial x}(2,3) = 4$, and $\frac{\partial p}{\partial y}(2,3) = 5$, estimate $p(2.1, 2.8)$.

a. 2.4 Correct Choice

b. 2.6

c. 2.8

d. 3.2

e. 3.4

$$p_{\tan}(x,y) = p(2,3) + \frac{\partial p}{\partial x}(2,3)(x-2) + \frac{\partial p}{\partial y}(2,3)(y-3) = 3 + 4(x-2) + 5(y-3)$$

$$p(3.2,1.9) \approx p_{\tan}(2.1,2.8) = 3 + 4(2.1-2) + 5(2.8-3) = 3 + 4(.1) + 5(-.2) = 2.4$$

4. If the temperature in a room is given by $T = 75 + xyz$ and a fly is located at $(2,1,4)$, in what unit vector direction should the fly fly in order to decrease the temperature as fast as possible?

a. $\frac{1}{\sqrt{21}}\langle 2,4,1\rangle$

b. $\frac{1}{\sqrt{21}}\langle -2,-4,-1\rangle$ Correct Choice

c. $\langle 4,8,2\rangle$

d. $\langle -4,-8,-2\rangle$

e. $\frac{1}{\sqrt{21}}\langle 2,-4,1\rangle$

$\vec{\nabla} T = \langle yz, xz, xy\rangle$ $\quad \vec{v} = \vec{\nabla} T\big|_{(2,1,4)} = \langle 4,8,2\rangle$ $\quad |\vec{v}| = \sqrt{16+64+4} = \sqrt{84} = 2\sqrt{21}$

Direction of Max increase is $\hat{v} = \frac{\vec{v}}{|\vec{v}|} = \frac{1}{\sqrt{21}}\langle 2,4,1\rangle$.

Direction of Max decrease is $-\hat{v} = \frac{-1}{\sqrt{21}}\langle 2,4,1\rangle$.

5. Find the equation of the plane tangent to the surface $x^2z^2 + xy^3 = 31$ at the point $(1,3,2)$. Its z-intercept is:

a. -31

b. 124

c. 120

d. 31 Correct Choice

e. 4

$P = (1,3,2)$ $\quad F = x^2z^2 + xy^3$ $\quad \vec{\nabla} F = \langle 2xz^2 + y^3, 3xy^2, 2x^2z\rangle$

$\vec{N} = \vec{\nabla} F\big|_P = \langle 2\cdot 1\cdot 4 + 27,\ 3\cdot 1\cdot 9,\ 2\cdot 1\cdot 2\rangle = \langle 35,27,4\rangle$

Tangent plane is $\vec{N}\cdot X = \vec{N}\cdot P$ or $35x + 27y + 4z = 35\cdot 1 + 27\cdot 3 + 4\cdot 2 = 124$

or $z = 31 - \frac{35}{4}x - \frac{27}{4}y$ The z-intercept is 31.