Name________________________	ID______________	
MATH 251	Quiz 5	Fall 2006
Sections 507	Solutions	P. Yasskin

1-3	/15
4	/10
Total	/25

5 points each

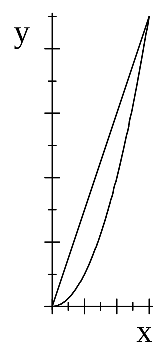
1. Compute $\int_0^1 \int_0^{z^2} \int_0^{xz} 30yz\,dy\,dx\,dz$.

a. 15
b. $\frac{1}{4}$
c. $\frac{2}{3}$
d. $\frac{5}{3}$
e. $\frac{1}{2}$ Correct Choice

$$\int_0^1 \int_0^{z^2} \int_0^{xz} 30yz\,dy\,dx\,dz = \int_0^1 \int_0^{z^2} \left[15y^2z\right]_{y=0}^{xz} dx\,dz = \int_0^1 \int_0^{z^2} 15x^2z^3\,dx\,dz$$
$$= \int_0^1 \left[5x^3z^3\right]_{x=0}^{z^2} dz = \int_0^1 5z^9\,dz = \left[\frac{5z^{10}}{10}\right]_{z=0}^{1} = \frac{1}{2}$$

2. Find the volume of the solid below $z = 2xy$ above the region in the xy-plane between $y = 3x$ and $y = x^2$.

a. $\frac{729}{2}$
b. $\frac{243}{4}$ Correct Choice
c. $\frac{81}{8}$
d. $\frac{243}{2}$
e. $\frac{81}{4}$



$$\int_0^3 \int_{x^2}^{3x} 2xy\,dy\,dx = \int_0^3 \left[xy^2\right]_{y=x^2}^{3x} dx = \int_0^3 [9x^3 - x^5]\,dx = \left[\frac{9}{4}x^4 - \frac{x^6}{6}\right]_0^3 = 3^6\left[\frac{1}{4} - \frac{1}{6}\right] = \frac{243}{4}$$

3. Reverse the order of integration in the integral $\int_0^2 \int_0^{y^2} \sqrt{x^4 + y^3}\, dx\, dy$

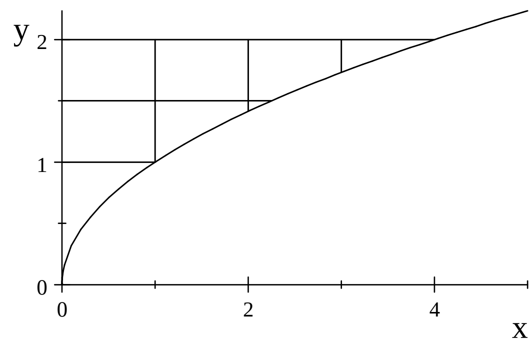
a. $\int_0^2 \int_0^{x^2} \sqrt{y^4 + x^3}\, dy\, dx$

b. $\int_0^2 \int_0^{x^2} \sqrt{x^4 + y^3}\, dy\, dx$

c. $\int_0^4 \int_{\sqrt{x}}^2 \sqrt{x^4 + y^3}\, dy\, dx$ Correct Choice

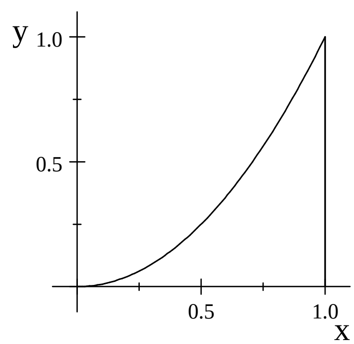
d. $\int_0^4 \int_0^{\sqrt{x}} \sqrt{x^4 + y^3}\, dy\, dx$

e. $\int_0^{y^2} \int_0^2 \sqrt{x^4 + y^3}\, dy\, dx$



The original order says: For each y between 0 and 2, we have $0 \le x \le y^2$.
The new order says: For each x between 0 and 4, we have $\sqrt{x} \le y \le 2$.

4. (10 points) Find the mass and center of mass of the region between $y = 0$, $y = x^2$ and $x = 1$, if the area density is $\rho = xy$.



$$M = \int_0^1 \int_0^{x^2} xy\, dy\, dx = \int_0^1 \left[\frac{xy^2}{2} \right]_{y=0}^{x^2} dx = \int_0^1 \frac{x^5}{2}\, dx = \frac{x^6}{12} \Big|_0^1 = \frac{1}{12}$$

$$x\text{-mom} = M_y = \int_0^1 \int_0^{x^2} x^2 y\, dy\, dx = \int_0^1 \left[\frac{x^2 y^2}{2} \right]_{y=0}^{x^2} dx = \int_0^1 \frac{x^6}{2}\, dx = \frac{x^7}{14} \Big|_0^1 = \frac{1}{14}$$

$$y\text{-mom} = M_x = \int_0^1 \int_0^{x^2} xy^2\, dy\, dx = \int_0^1 \left[\frac{xy^3}{3} \right]_{y=0}^{x^2} dx = \int_0^1 \frac{x^7}{3}\, dx = \frac{x^8}{24} \Big|_0^1 = \frac{1}{24}$$

$$\bar{x} = \frac{x\text{-mom}}{M} = \frac{M_y}{M} = \frac{1}{14} \cdot \frac{12}{1} = \frac{6}{7}$$

$$\bar{y} = \frac{y\text{-mom}}{M} = \frac{M_x}{M} = \frac{1}{24} \cdot \frac{12}{1} = \frac{1}{2}$$