M442 Fall 2017, Assignment 1, due Friday Sept. 8

1. [10 pts] For each of the parts below, print out a diary session of your MATLAB commands, and also print out any figures you generate.

a. **Calculations**

Use MATLAB to make the following calculations.

a.1. $r = \sqrt{1 - \frac{2}{\pi^5}}$.

a.2. $r = e^2 \ln 5$. (MATLAB's convention is log() for natural log.)

a.3. $r = \sin^2 2 + \cos^2 4$, 2 and 4 measured in radians.

a.4. r = $\sin^2 30 + \cos^2 40$, 30 and 40 measured in degrees.

b. **Algebra**

Use MATLAB to solve the following algebraic equations.

b.1. $x^3 + 3x^2 + 2x - 1 = 0$.

b.2. $\begin{aligned} 2x + 3y &= 0 \\ x + y &= 1. \end{aligned}$

c. **Calculus**

Use MATLAB to solve the following calculus problems.

c.1. Find the point $x_{\max}$ at which the function

$$f(x) = \frac{3 - e^{x^2}}{\sqrt{1 + x^2}} + 5x + 8$$

is globally maximized. (Your solution should be accurate to at least two decimal places.)

c.2. Compute $\int_1^2 e^{-x^2} dx$.

c.3. Plot $f(x) = e^{-x^2}$ for $x \in [-5, 5]$ and use MATLAB's built-in command *area.m* to indicate the area under the graph that you computed in (c.2).

d. **Matrix operations**

Use MATLAB to solve the following matrix problems.

For $X = [1, 2, 3, 4]$ and $Y = [5, 6, 7, 8]^T$, where T indicates transposition so that Y is a column vector, use MATLAB to compute

d.1. XY

d.2. YX

d.3. The vector containing as its first entry the first element of X multiplied by the first element in Y, as its second entry the second element of X multiplied by the second element in Y etc.

2. [10 pts] For these plotting assignments, use *plot* rather than *ezplot* or *fplot*. You should write a single MATLAB script M-file that produces all your plots, and you should turn this in along with a separate plot for each part. If you put the *figure* command above each plot command MATLAB will create your five plots in five different windows, and you will have access to them all at once.

a. Plot the function $f(x) = x + \sin x$ for $x \in [0, 2\pi]$. Label your x and y axes and add a title to your plot.

b. Plot the functions $x(t) = \tan t$ and $y(t) = \cos t$ on a single figure for $t \in (-\frac{\pi}{4}, \frac{\pi}{4})$. Label your x and y axes (i.e., your horizontal and vertical axes, since your horizontal variable will be t) and add a title to your plot. Determine (roughly) from your figure the point of intersection.

c. Plot the path $(x(t), y(t))$ for $x(t)$ and $y(t)$ as defined in (b), again for $t \in (-\frac{\pi}{4}, \frac{\pi}{4})$. Label your x and y axes and add a title to your plot.

d. Use MATLAB's built-in help to find information about the *legend* function, and use this function to add a legend to your plot from (b).

e. MATLAB's built-in functions *text* and *gtext* can both be used to add text to a plot at a particular location. Use the command *text(0,-.2,'tan(t)')* to label the curve $x(t) = \tan t$ on your plot from (b). (This command places the text *tan(t)* so that its first character is at the designated position $(0, -.2)$). Likewise, use the command *gtext('cos(t)')* to place the text *cos(t)* in a reasonable location relative to the curve $y(t) = \cos t$. (This command will open up your figure window and allow you to select the location of the text with a mouse click.)

3. [10 pts] In this problem, we'll recall a few basic concepts from finance. First, if some principal investment P is invested for t years at interest rate r, then if the interest is compounded annually, the return is $R_t = P(1 + r)^t$. If the interest is compounded twice a year (and at even intervals) the return is $R_t = P(1+\frac{r}{2})^{2t}$. That is, the interest rate is cut in half over each period, but there are twice as many periods. More generally, if the interest is compounded n times a year (again, at evenly spaced intervals) the return becomes $R_t = P(1 + \frac{r}{n})^{nt}$. Recalling that $\lim_{n\to\infty}(1 + \frac{1}{n})^n = e$, we see that continuously compounded interest will yield a return governed by $R_t = Pe^{rt}$. ($\lim_{n\to\infty}(1 + \frac{r}{n})^{nt} = \lim_{\frac{n}{r}\to\infty}((1 + \frac{r}{n})^{\frac{n}{r}})^{rt} = e^{rt}$.)

a. Write a MATLAB script M-file that computes the return after 5 years for an investment of 100 dollars at 5% ($r = .05$) under three different plans: compounding annually, compounding monthly, and compounding continuously. Include both the M-file and a diary session of its implementation.

b. Write a MATLAB function M-file that takes as input (in the function statement) t years, interest rate r, and principal P, and returns values through the function statement from the same three plans as described in (a). Include both the M-file and a diary session of its implementation.

4. [10 pts] An amortized loan is one in which an initial principal P_0 is borrowed at some interest rate r, and payments with value v are made at equal time intervals, often monthly. (For example, car and house payments typically work this way.) Here, we are assuming r denotes interest rate for the payment period, so for example if the annual interest rate is .05 and the payments are monthly, then $r = .05/12$. (To be clear, it's generally assumed in these situations that when we say annual rate r_A we mean monthly rate $\frac{r_A}{12}$, even though monthly growth at rate $\frac{r_A}{12}$ isn't quite the same as annual growth at rate r_A.) If we let P_t denote the remaining balance at time t, then we clearly have the difference equation

$$P_{t+1} = P_t(1 + r) - v,$$

which can be solved exactly as

$$P_t = (1+r)^t P_0 - v\frac{(1+r)^t - 1}{r}.$$

a. Write a function M-file that takes as input the values r and P_0, and the duration of the loan, and returns the payment value v required to pay off the loan.

b. If a car loan is $P_0 = 20,000$ for five years at annual rate .05, what must the monthly payment v be?

5. [10 pts] In this problem we will consider MATLAB objects known as *anonymous* functions, and also a number of additional plot commands. You should write a single MATLAB script M-file that produces all your plots, and you should turn this in along with a separate plot for each part.

a. Define the function

$$f(x) = x^2 e^{-x^2}$$

by using the command *f=@(x)x^2*exp(-x^2)*. Technically, f is now a *function handle*, which is an easy object to work with. For example, you can now evaluate f in the MATLAB Command Window by typing $f(0)$, $f(1)$ etc. Plot this function on the interval $[0, 2]$ by using the command *fplot(f,[0,2])*.

b. For implicit relations such as $x^2 + y^2 = 1$, plotting with either *plot* or *fplot* can be clunky, and it's often more convenient to use *ezplot*. As an example, we'll plot the *Folium of Descartes*

$$x^3 + y^3 = 6xy.$$

First, we re-write the equation as $x^3 + y^3 - 6xy = 0$ and define the left-hand side as an anonymous function *f=@(x,y)x.^3+y.^3-6*x.*y*. Now, plot the Folium for $x \in [-1, 4]$ with the command *ezplot(f,[-1,4])*. (Notice that MATLAB automatically sets $f = 0$.)

c. Use MATLAB's *plot3* command to plot the helix described parametrically by

$$\begin{aligned} x(t) &= \cos t \\ y(t) &= \sin t \\ z(t) &= t, \end{aligned}$$

for $t \in [0, 12\pi]$.

d. Use MATLAB's *mesh* command to plot the function

$$f(x, y) = x^2 + y^2$$

on the square $[-2, 2] \times [-2, 2]$. (**Hint.** You'll find MATLAB's help on the *meshgrid* command useful for this problem.)