

Ballistics Project

Due Friday, November 10

M442, Fall 2017

1 Overview

Ballistics is the science of projectile motion and impact, phenomena well described by Newtonian mechanics. The number of applications of this type of analysis is staggering, ranging from such mundane issues as automobile accident simulations and optimal golfing to the critical studies of missile defense and space exploration. Somewhat less dramatically, in this project we will use Newtonian mechanics to describe the flight of a sponge dart, light enough so that air resistance will play a critical role.

2 Experimental data

The data for this project was collected by firing sponge darts from a toy gun (\$3.99, Wal-Mart—it's not all that easy talking the department into financing this sort of thing). First, I tabulated a set of measurements for distance traveled (by the projectile) versus angle of inclination of the gun, taking angles of inclination 5, 10, 15, ..., 85 degrees. The darts were fired from a height of .18 meters.

Angle of inclination	5	10	15	20	25	30	35	40	45
Distance traveled	4.37	5.23	6.95	7.84	8.17	8.69	8.81	8.99	8.95
Angle of inclination	50	55	60	65	70	75	80	85	
Distance traveled	8.83	8.19	7.84	7.12	6.38	5.08	3.34	2.13	

Next, I computed the time it took for a dart fired straight up from a height of .39 meters to hit the ground: 2.13 seconds. Finally, I computed the time it took one of the darts to fall 4.06 meters: .95 seconds. These fairly simple measurements will suffice for the assignments in this project.

For the assignments in Sections 3.1, 3.2, and 3.3, models will be based entirely on these final two experiments, and the table of angles of inclination and distances traveled will only be used for evaluation of the model. The assignment in Section 3.4 will address using the entirety of our data in the development of a model.

3 Assignments

3.1 Analysis in the absence of air resistance

Ignoring air resistance, Newton's equations of motion for an object under the influence of gravity alone are quite straightforward. Letting y represent the height of the object and x represent its distance traveled, we have¹

$$\frac{d^2}{dt^2}y(t) = -g, \quad \frac{d^2}{dt^2}x(t) = 0. \quad (1)$$

1. Use the experimental data from Section 2 to determine the initial velocity with which the darts are fired.
2. Solve these two second order differential equations (1) for $x(t)$ and $y(t)$ and use MATLAB to plot the (x, y) trajectory obtained for angle of inclination $\theta = 35^\circ$. Observe that $x(0)$ and $y(0)$ were determined by the set-up of my measurements and are known to be 0 and .18 m respectively. We will see in class that the remaining two initial conditions $x'(0)$ and $y'(0)$ are given by

$$x'(0) = v \cos \theta, \quad y'(0) = v \sin \theta,$$

where v is the velocity with which the gun fires its darts.

3. Determine the distance, $d(\theta)$, your object travels as a function of θ , and compute the angle that maximizes its distance. Your answer should be in degrees and accurate to two decimal places. Plot this function on MATLAB and compare it with my experimental values. Discuss the discrepancies.
4. Compute an error, E , for your model based on the sum of squared errors,

$$E = \sqrt{\sum_{k=1}^{17} (d_{\text{experimental}} - d_{\text{model}})^2}.$$

3.2 Linear air resistance

A typical first attempt at introducing the effects of air resistance into a physical model is through linear air resistance, terms of the form $-bx'(t)$. Adding such terms to (1), we obtain

$$\frac{d^2}{dt^2}y(t) = -g - b \frac{d}{dt}y(t), \quad \frac{d^2}{dt^2}x(t) = -b \frac{d}{dt}x(t), \quad (2)$$

with the same initial conditions as before. We will now work through steps similar to those in the case of no air resistance, except that as the mathematics becomes more cumbersome, we will begin to fall back more on MATLAB.

1. Develop an analytic solution to equations (2).

¹We're also ignoring the fact that the gravitational pull on an object above the earth depends upon its height above the earth. We take $g = 9.81m/s^2$.

2. Use your solution to equations (2) and the experimental data from Section 2 to determine the value of b for this model.
3. Find a (corrected) initial velocity v .
4. Plot this trajectory (y versus x) for angle $\theta = 35^\circ$ along with your similar trajectory in the absence of air resistance.
5. Analytically, determine an implicit equation for $d(\theta)$, the distance the dart travels given that it was shot with angle of inclination θ . Determine the value of θ that maximizes $d(\theta)$.
6. Plot the three different versions of $d(\theta)$ —experimental data, no air resistance data, and linear air resistance data—on the same figure, and again discuss the discrepancies.
7. Compute your error E for this model.

3.3 Physical air resistance

In general, linear air resistance is not an adequate model of projectile motion. In this part of the project, you will work through the steps of the previous cases with a more physical form of air resistance. Unfortunately, analytic solutions for your model will be too cumbersome to derive or work with, so you will have to do most of your analysis with MATLAB.

1. Use the method of dimensional analysis to determine a more physical form for your air resistance.
2. Use MATLAB and the experimental data from Section 2 to determine the value of b for your model.
3. Use your new value for b to determine a (further corrected?) initial velocity v .
4. Use MATLAB's *event location* to plot a trajectory for angle $\theta = 35^\circ$, and plot it along with your two previous trajectories at this angle.
5. Plot $d(\theta)$ for this model along with $d(\theta)$ from the previous sections. Determine the value of θ that maximizes $d(\theta)$ for this model.
6. Compute your error E for this model and compare it with your errors from the previous models.

3.4 Regression Approach

If we have the same amount of confidence in each measurement, we expect to obtain better values for b and $|\vec{v}_0|$ by using all the data in our table.

1. Using the values of b and v from Section 3.3 as initial guesses, carry out a nonlinear least squares regression fit for the function $D(\theta; |\vec{v}_0|, b)$, taken to return the horizontal distance traveled by a dart launched with angle of inclination θ , initial velocity v , and coefficient of air resistance b . You should obtain yet further corrected values for b and v .
2. Plot your fit for $D(\theta; |\vec{v}_0|, b)$ along with your plots of $d(\theta)$ from the previous three cases.
3. Compute our standard error estimate E for this fit.