Portfolio Theory Project

M442, Spring 2003

1 Overview

On October 22, 1929, Yale economics professor Irving Fisher was quoted in a New York *Times* article entitled "Fisher says prices of stocks are low," that during the next few weeks there would be, "a ragged market, returning eventually to further steady increases." On October 24, 1929, *Black Thursday,* the New York stock exchange lost approximately four billion dollars, roughly 3.1% of its value.[1] The drop was even worse on Tuesday, October 29, 1929, *Black Tuesday*, when the NYSE lost approximately five billion dollars, or 4% of its (remaining) value. The crash had begun.

Could any form of mathematical modeling have predicted the stock market's tumultuous behavior in the fall of 1929? Almost certainly not. The fact of the matter is, however, that many market analysts employ a great deal of mathematics in determining their portfolio decisions. In this project, you will study the fundamental tools these analysts employ, and develop your own model of stock and portfolio behavior.

2 Investment Basics

In this section, we will consider a few concepts from the theory of finance. Though in no way comprehensive, the list should serve to get you started thinking about the types of things that stock price models should include.

2.1 Common Stock

Had you managed to scrape together, say $20,000, in July 1995 and invested it in Dell computers (DELL), and additionally been savvy enough to sell in February 1999, you would be retired now, worth 1.1 million dollars. When most of us think of owning stock, this is the type of stock that comes to mind, *common stock*—a share of a company, available to the general public. Common stock is the investment that will concern us in this project.

In general, each share of stock represents a partial ownership of the company, and consequently rights to some percentage of the company's net profits. For example, if company XYZ issues 100 shares of stock, and an investor buys 1, then the investor has a limited ownership of 1% of the company and is entitled to 1% of the company's net profit.[2] Suppose the company expects to make $1,000 over the next year. Then an investor might consider $10 a fair price per share: the amount he will earn per share in one year (1% of $1,000). Suppose, however, that the company expects to make $1,000 per year for the next five years. In that case, an investor might be willing to pay $20 or even $30 per share, confident that even though it will take two or three years for him to recover his initial investment, he will make a solid profit afterwards. (Of course, the company could also go bankrupt, and he could lose his entire investment.) At this point, the price of the stock is determined like the price of most other commodities in a capitalistic society, by what people are willing to pay. Of course, the factors influencing this willingness to pay are far too complicated to fully understand, and therefore to some extent stock prices behave randomly.

[1] An article in the October 26, 1929, New York *Times* was entitled, "Caution Advised by Stock Brokers." Thanks for the tip, guys.

[2] The limits of ownership are both good and bad. If you own stock in a company but don't actively participate in running the company, then you cannot be personally found liable if the company is sued. On the other hand, you don't get to actively participate in running the company.

2.1.1 Price-earnings multiples

A number critical in the valuation of stocks is the *price-earnings multiple* (P/E):

$$\text{P/E}=\frac{\text{Price of one share of common stock}}{\text{Company earnings per share of common stock}}=\frac{\text{S}}{\text{EPS}}.$$

If company XYZ has current earnings \$1,000, with 100 shares issued, then its earnings per share is \$10. If the current market value of XYZ stock is \$30, then the stock's price-earnings multiple is $\frac{30}{10}$ =3. Price-earnings multiples tend to vary widely both from company to company and across industries. A quick perusal of Yahoo.com's finance page reveals that the P/E for Wal-Mart (WMT) is currently 29.4, while the P/E for Kroger's (KR) is 8.9. Typically, we associate a high P/E with an overvalued stock (and consequently a poor investment), and a low P/E with an undervalued stock (and consequently a good investment). The price-earnings multiples for Internet stocks during the boom in the nineties were staggering. In fact, many start-ups like Amazon (AMZN) didn't have earnings (just mounting debt), so the P/E couldn't even be calculated (or could be considered infinite).

2.1.2 Risk Assessment and Modern Portfolio Theory

If you invest \$100 in a savings account at 2% interest for a year, then at the end of the year, barring incalcuably improbable disasters, you will have \$102. If, on the other hand, you invest the \$100 in a corporation, you might end up invested in the next Dell, and you might end up invested in the next Enron. For this reason, we consider stocks to be riskier investment instruments than savings accounts. They are also, by and large, signifigantly more lucrative.

Modern Portfolio Theory (MPT) assumes that investors expect to be compensated in some way for taking on greater risk. The idea is simply that if two investments offer the same expected return, the educated investor will choose the less risky of the two. One fairly simple (though controversial) measure of risk is the *beta* value (β), which in theory describes the volatility of a stock *relative to the market*. There are two equivalent formulations of β:

1. **Academic Formulation.** Let R_S represent the vector of percent returns on some stock S as measured over some specified period of time. For example, we might consider the percent return each month for 60 months. Suppose the value of the stock in Month k is 15 and in Month $k+1$, 20. Then

   $$R_{S_k}=\frac{20-15}{15}=\frac{1}{3},$$

   roughly a 33% return on investment that month. The 59 other components of R_S would be calculated similarly. For some market index—the Dow Jones Industrial Average, the Nasdaq, the New York Stock Exchange, the S&P 500, or any other reasonable measure—compute the vector of percent returns R_M. We define the beta value (β) of the stock S by the relation

   $$\beta=\frac{\text{Cov}(R_S,R_M)}{\text{Var}(R_M)}.$$

2. **MPT Formulation.** An equivalent, though perhaps more illuminating formulation of beta is described through,

   $$R_S=r+\beta(R_M-r),$$

   where r is the risk-free investment rate (i.e., the best guaranteed return we can get on our money). Very generally, the market return R_M is greater than the risk-free return r (otherwise Federal Reserve Board Chairman Alan Greenspan will (effectively) lower it). This means that in the event that $\beta \leq 1$, $R_S \leq R_M$ and we consider stock S less risky (or more stable) than the general market. On the other hand, if $\beta > 1$, then $R_S > R_M$ and we consider stock S to be more risky (or less stable) than the general market.

2.1.3 Stock Dividends

The primary way in which we think of money being made on the stock market is as in the example of Dell computers, by the price of the stock rising dramatically over a relatively short period of time. This is referred

to as *capital gain* (or *capital loss*, as the recent case has largely been). In addition to this, some stocks offer *dividends.* If a company generates more income than it requires for operation and expansion, it may elect to pay the excess to its shareholders. For example, Microsoft (MSFT) recently began paying a dividend of (hold onto your chair) $.08 per share, annually. (That is, if you own 100 shares of Microsoft, Bill Gates will cut you a check once a year for $100 \times .08 = \$8.00$.) (The rate at which dividends are paid varies, some quarterly, some monthly, some yearly.)

Analysts are divided on whether dividends make stocks more attractive or less. The argument against buying stocks that pay dividends is that under current tax laws dividends are an inefficient way to distribute earnings.[3] Suppose, for instance, the net income for company XYZ is roughly $1 per share greater than the amount it requires to operate and expand, and the board of directors decides to pay the excess to shareholders. First, Uncle Sam levies a corporate tax on this income of, say, 25%, so that $.75 remains for distribution. Each shareholder is then paid $.75 per share, which is subject to individual income tax, say 30%. In the end, the shareholder gets $.53 out of the original buck.

The argument in favor of buying stocks that pay dividends is two fold. First, a company only pays dividends if it is not only making money hand over fist, but is confident that it will continue to do so for years to come. A dividend is a very strong statement regarding a company's future earnings. Second, dividends constitute a steady stream of income that remains independent of share price.

2.2 The Efficient-Market Theory

Often referred to as the *academic* theory and occasionally (by academics) as the *random-walk* theory, the efficient-market theory of stock valuation contends that everything that can be known about the value of a stock is reflected in the stock's current price. The idea behind this theory is that with so many investors constantly evaluating stocks, if any stock price falls even marginally off target, a rash of buys or sells will force its adjustment. Thus all stock prices, according to this theory, are fair, and in particular, no stock looks better or worse than any other. Sure, some stocks are Honda Civics and some Jaguars, but the difference in quality is reflected in the difference in price. The Nobel laureate Paul Samuelson explained it as follows [S]:

> If intelligent people are constantly shopping around for good value, selling those stocks they think will turn out to be overvalued and buying those they expect are now undervalued, the result of this action by intelligent investors will be to have existing stock prices already have discounted in them an allowance for their future prospects. Hence, to the passive investor, who does not himself search out for under- and overvalued situations, there will be presented a pattern of stock prices that makes one stock about as good or bad a buy as any other. To that passive investor, chance alone would be as good a method of selection as anything else.

In 1967, *Forbes* magazine began testing this idea by choosing a portfolio by throwing darts at the stock market page of the New York *Times*. In 1984, when they concluded the study, their initial (theoretical) investment of $28,000 in 28 stocks had grown to $131,697.61, a 9.5% average annual return—better than most professionally managed accounts over the same period.

For our purposes, the critical point will be that *according to the efficient-market theory the rate of change in time of a stock depends only on the current value of the stock.*

3 Stock Price Analysis

In his book, *A Random Walk Down Wall Street*, Burton G. Malkiel writes,

> Finally, there is the enormous difficulty of translating known information about a stock into an estimate of true value. We have seen that the determinants of a stock's value concern the extent and duration of its growth path far into the future. Estimating this is extraordinarily difficult, and there is considerable scope for an individual with superior intellect and judgment to turn in a superior performance.

In this section, we will take a very brief look at the three fundamental schools of thought regarding the analysis of stock prices.

[3] But keep your eye on the current executive budget, which suggests changing this paradigm.

3.1 Technical Analysis

In finance jargon, *technical analysis* or *charting* refers to the method of trying to determine the future progression of a stock price by studying past behavior, both of that stock and of other similar stocks. A technical analyst's chart typically consists of a series of vertical lines, one for each time period (days, weeks, months, years) stretching from the stock's low value over the period to the stock's high value over the period, with a mark at its closing price. As an example, consider the value of Microsoft's stock over the 5-year or 61-month period, January 1, 1997 through January 2, 2002. First, for the month of December, 1996 (the data from January 1, 1997) Microsoft's (split- and dividend-adjusted) low was 10.06, its high 12.86, and its close 12.71. The bar associated with this month alone is shown in Figure 1. A 5-year (61-month) chart for Microsoft in this period is shown in Figure 2. (Technical charts are extremely easy to create by downloading data in spreadsheet format and then uploading it into MATLAB.)

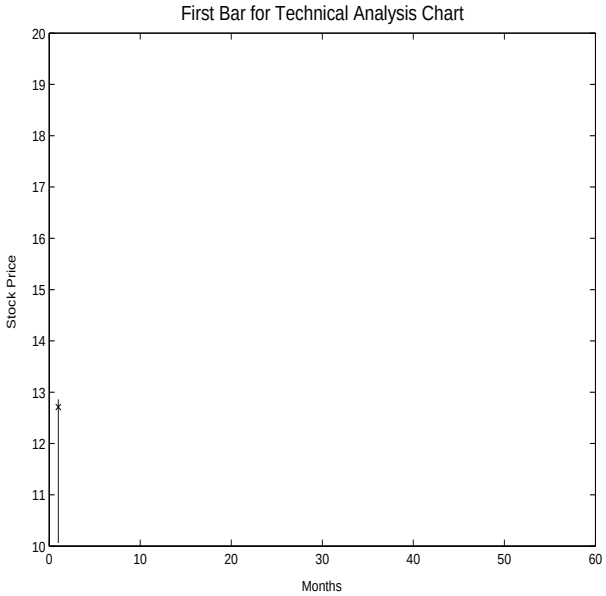

Figure 1: Single bar for a technical analysis chart.

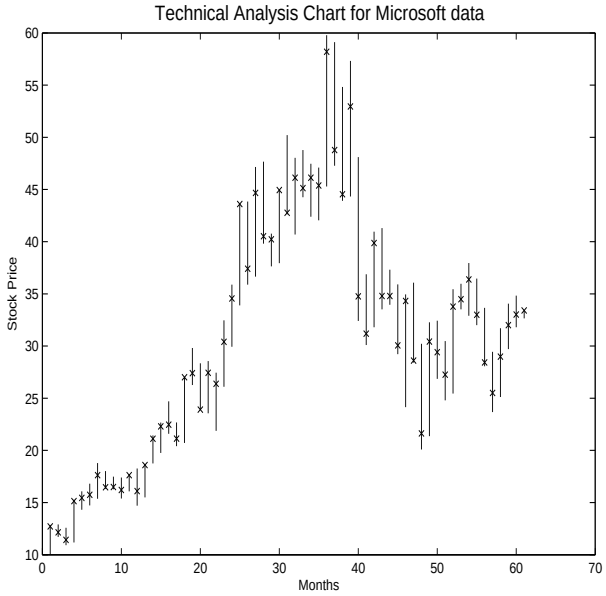

Figure 2: Technical Chart for Microsoft, January 1, 1997–January 2, 2002.

Though technical analysts can be long-winded about explaining what they do, the idea behind charting is simply to look for patterns in behavior, both within the path of the stock under consideration and in all paths across industries.

3.2 Fundamental Analysis

The *firm-foundation* theory of stock pricing holds that each stock has an inherent worth—based on company assets, dividend product demand, sector pricing, etc.—that may or may not be accurately reflected in its price. The idea of *fundamental analysis* is to determine this intrinsic worth, and to buy stocks that are priced below it. In general, what fundamental analysts want to predict is future earnings, from which they suspect the price of the company's stock should be fairly evident.

3.3 The Efficient-Market Theory

As the efficient-market theory is our starting point for the GBM model, I've already discussed it in Section 2.2. The point I'll add here is that even assuming the efficient-market theory, GBM isn't the only possible model that could be employed to model stock prices. All the efficient-market theory states is that dS_t does not depend explicity on t.

4 Assignments

For this project, your group will carry out two analyses. First, you will study a model of stock price valuation often used in current market analysis, and second, you will develop your own model of stock price valuation.

4.1 Week 1 Assignment, soft deadline 4/11/03

1. Each member of your group should choose a stock to analyze and download 61 months of (monthly) data for that stock. Use the period January 1, 1997 through January 2, 2002. Since we don't have time to test our models on legitimately future market behavior, we will treat the period February 1, 2002 through April 1, 2003 as unknown.

4.2 Week 2 Assignment, soft deadline 4/18/03

For each of these assignments, use the period of data collected in Week 1. Each plot should be included in your final report.

1. Using our class file *technical.m*, Create a technical analysis chart for each of the stocks your group selected in Week 1.

2. Compute the value of β for each of the stocks your group selected in Week 1.

3. Develop a GBM model for each of the stocks your group selected in Week 1. Plot three sample paths for each of your models along with the actual data.

4. Use your GBM models to compute the expected value and variance for each of your stocks on April 1, 2003, and compare your expectations with the actual stock values on that day.

5. Compute the expected value and variance of your portfolio (as of April 1, 2003), and compare your expectations with the actual value of the entire portfolio. We have seen in class that expected value is linear; that is, that the expected value of your portfolio is simply the sum of expected values of your individual stocks. Critically, this is not the case with variance. How does the variance of your portfolio relate to the sum of variances of your individual stocks?

4.3 Week 3 Assignment, soft deadline 4/25/03

1. Building on the GBM model, develop a new model for each of your stocks, customized in at least one way toward that stock. Your final report should include a detailed explanation of any terms you add to the GBM model. Plot three sample paths for each of your new models along with the actual data.

2. Use your new model to compute a revised expected value and variance for each of your stocks, again on April 1, 2003, and compare your expectations both with the expectations of the GBM model and with the actual stock values.

3. Use your new model to compute the expected value and variance of your portfolio, and compare your expectation with both the GBM expectation of your portfolio and the actual value of the stock.

References

[M] B. G. Malkiel, *A Random Walk Down Wall Street: The Best Investment Advice for the New Century*, W.W. Norton & Co. 1999.

[S] P. A. Samuelson, statement before *Committee on Banking and Currency*, U. S. Senate, August 2, 1967.

[V] S. R. Veale, *Stocks Bonds Options Futures*, 2nd edition, New York Institute of Finance Press, 2001.