M469 Spring 2020, Assignment 10, due Friday, Apr. 24

Suggested Reading. *Controlling chaos in the brain*, by Steven J. Schiff et al, in *Nature* **370** (1994) 615-620. The study described in this article involves both the chaotic dynamics we discussed regarding nonlinear difference equations and the phase plane dynamics we will discuss for nonlinear differential equations. (I don't think the phase plane is particularly useful for difference equations, so I omitted it from our discussion. If you're curious, you can find a brief (and arguably misleading) discussion of the phase plane in difference equations in our reference by Allman and Rhodes.) The object of study in this article is neuronal activity associated with epilepsy. Working with slices from the hippocampus of the temporal lobe of rat brains, the authors induced epileptic type behavior by injection with convulsant drugs that reduce neuronal inhibition. They then measured the electrical activity associated with the resulting neuronal firing and looked for signs of chaotic behavior. This behavior is well-described in the first paragraph of their section *Chaos Identification and Control*. Notice particularly the eigenvectors that emerge experimentally in their phase plane diagram in Figure 2b. Finally, they are interested in controlling this chaotic behavior by introducing electrical pulses into the system that return the system to the stable manifold associated with the saddle fixed points they observe.

1. [10 pts] If an Allee effect is incorporated into the logistic model we obtain the equation

$$\frac{dy}{dt} = ry(1 - \frac{y}{K})(\frac{y}{L} - 1),$$

where L denotes a population threshold such that if the population ever decreases below L we expect it to die off. The parameters satisfy $r > 0$ and $0 < L < K$. Non-dimensionalize this system, and then find all the equilibrium points for your non-dimensionalized equation and determine the parameter ranges over which they are asymptotically stable and the parameter ranges over which they are unstable.

2. [10 pts] Suppose that in the absence of fishermen the population of fish, $y(t)$, in a certain lake follows a logistic model, and that fishing yield is added as a percent of population. Determine the maximum sustainable yield for this population of fish and describe what will happen to the fish population if the maximum sustainable yield is harvested.

3. [10 pts] Suppose fishing is suspended in the situation described in Problem 2. Compute the recovery time to the original carrying capacity from the population associated with maximum yield. Your recovery time will depend on r.

4. [10 pts] Solve the linear system of differential equations

$$\frac{dy_1}{dt} = 5y_1 + 2y_2; \quad y_1(0) = 0$$
$$\frac{dy_2}{dt} = 2y_1 + 2y_2; \quad y_2(0) = 1.$$