M469 Spring 2020, Assignment 6, due Fri., Feb. 28

Suggested Reading 1. Chapter 2 in *Mathematical Models in Biology: An Introduction*, by E. S. Allman and J. A. Rhodes. This is a chapter on age-structured populations, and it contains most of the material we've been discussing in class, including Leslie and Usher models, a brief review of eigenvalues and eigenvectors, and one of our approaches to solving a linear systems of difference equations.

Suggested Reading 2. Chapters 4 and 5 (on constructing phylogenetic trees) in the book mentioned above. Chapters 4 and 5 are by far the best in this book. If you want to better understand why it's so hard to find a textbook for this entire course, read the authors' wildly inadequate discussion of single difference equations in Chapter 1.

1. [10 pts] For the Allee type delay difference equation

$$y_{t+1} - y_t = a y_t (1 - y_{y-1})(y_{t-1} - b),$$

with $a > 0$, $0 < b < 1$, find the fixed points and determine values of a and b for which each is stable.

2. [10 pts] Write the following delay equations as first order systems:

a.
$$y_{t+1} = y_t e^{r(1 - \frac{y_{t-1}}{K})}.$$

b.
$$y_{t+1} = y_t e^{r(1 - \frac{y_{t-3}}{K})}.$$

3. [10 pts] In this problem we'll compare a Leslie structured population model with an Usher population model for a population that we'll assume can be divided into three age classes. Let s_1, s_2, s_3, and b denote positive parameters, and consider the Leslie model $\vec{y}_{t+1} = L\vec{y}_t$, where

$$L = \begin{pmatrix} 0 & 0 & b \\ s_1 & 0 & 0 \\ 0 & s_2 & 0 \end{pmatrix},$$

along with the Usher model $\vec{y}_{t+1} = U\vec{y}_t$, where

$$U = \begin{pmatrix} 0 & 0 & b \\ s_1 & 0 & 0 \\ 0 & s_2 & s_3 \end{pmatrix}.$$

Draw a transition diagram for each matrix, and explain what process each parameter in these models corresponds with physically. Show that the Usher matrix has an eigenvalue larger than the largest eigenvalue of the Leslie matrix, and use this observation to explain why the Usher model predicts faster population growth.

Note. You won't have to find a closed form expression for the eigenvalues of the Usher matrix.

4. [10 pts] Suppose a certain forest is composed of two species of trees, A and B. Each year $\frac{1}{3}$ of the trees of species A are replaced by trees of species B, while $\frac{1}{4}$ of the trees of species B are replaced by the trees of species A. The remaining trees either survive or are replaced by trees of their own species. Draw a transition diagram for these populations, and write down a model for this situation. Solve your model for initial populations $A_0 = 25$ and $B_0 = 50$. What happens to the tree populations as t gets large?