On the stability of Scott-Zhang type operators and application to multilevel preconditioning in fractional diffusion

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In this talk, we consider locally $L^2(\Omega)$-stable operators mapping into spaces of continuous piecewise polynomial set on shape regular meshes with certain approximation properties in $L^2(\Omega)$. For such operators, we discuss the following stability results:

- These operators are stable mappings $H^{3/2}(\Omega) \to B^{3/2}_{2,\infty}(\Omega)$.
- If the mesh is additionally quasi-uniform, for the space of continuous piecewise polynomials on this mesh, we have a sharper stability estimate $B^{3/2}_{2,\infty}(\Omega) \to B^{3/2}_{2,\infty}(\Omega)$.

Given a mesh $\mathcal{T}$ obtained by *Newest Vertex Bisection* (NVB) refinement from a regular triangulation $\widehat{\mathcal{T}}_0$ and $\widehat{\mathcal{T}}_\ell$ as the sequence of uniformly refined NVB-generated meshes, we introduce the *finest common coarsening* (`fcc`) of two meshes $\widetilde{\mathcal{T}}_\ell := \texttt{fcc}(\mathcal{T}, \widehat{\mathcal{T}}_\ell)$. For the space of continuous piecewise polynomials defined on the mesh hierarchy $\widetilde{\mathcal{T}}_\ell$, we construct the modified Scott-Zhang operator $\widetilde{I}^{SZ}_\ell$ in such a way that for continuous piecewise polynomials on $\mathcal{T}$, this operator coincides with the Scott-Zhang operator $\widehat{I}^{SZ}_\ell$ on $\widehat{\mathcal{T}}_\ell$.

Since the Scott-Zhang operators are local, $L^2(\Omega)$-stable operators with certain approximation properties in $L^2(\Omega)$, therefore these operators admit the above stability results. Taking advantage of the stability results and the mentioned property of the modified Scott-Zhang operators, we present multilevel norm equivalences in the Besov spaces $B^{3\theta/2}_{2,q}(\Omega)$, $\theta \in (0,1)$, $q \in [1,\infty]$.

As an application, we present a local multilevel diagonal preconditioner for the integral fractional Laplacian $(-\Delta)^s$ for $s \in (0,1)$ on adaptively refined meshes and prove this multilevel diagonal scaling gives rise to uniformly bounded condition number for the integral fractional Laplacian. To prove the main result, we apply the norm equivalence of the multilevel decomposition.