General Heat Equation

$\vec{x} = (x_1, x_2, \ldots, x_n)$, $\underline{k=1}$, $-\infty < x < \infty$

When $\underline{n=1}$, we proved that the solution of

$$(1)\begin{cases} u_t = u_{xx}, & t > 0 \\ u(x,0) = f(x), & t = 0 \end{cases}$$

is $$u(x,t) = \int_{-\infty}^{\infty} f(\bar{x}) \frac{1}{\sqrt{4\pi t}} e^{-\frac{(x-\bar{x})^2}{4t}} d\bar{x}.$$

Properties of $G(x,t) = \dfrac{1}{\sqrt{4\pi t}} e^{-\frac{x^2}{4t}}$:

(a) $G(x,t) \geq 0 \quad \forall x, \ t > 0$

(b) $\displaystyle\int_{-\infty}^{\infty} G(x,t)\, dx = 1$

(c)* $\displaystyle\lim_{t \to 0} \int_{-\infty}^{\infty} f(\bar{x}) \frac{1}{\sqrt{4\pi t}} e^{-\frac{(x-\bar{x})^2}{4t}} d\bar{x} = f(x)$

$\forall x \in \mathbb{R}$

Now, we want to repeat this <u>for $n > 1$</u>. That is:

#1 We need to find $G(x,t)$ such that

(a) $G \geq 0$

(b) $\int G\, dx = 1$

(c) $u(x,t) = \displaystyle\int_{\mathbb{R}^n} f(\bar{x})\, G(x-\bar{x})\, d\bar{x}$ solves $\underline{(1)}^n$