3.4.1: $\int_a^b u \frac{dv}{dx} dx = uv \Big|_a^b - \int_a^b v \frac{du}{dx} dx.$

$u, v, \frac{du}{dv}, \frac{dv}{dx}$. continuous for $a \le x < c$. & $c \le x \le b$.

10 15 a) Derive an expression for $\int_a^b u \frac{dv}{dx} . dx$.

$$\int_a^b u \frac{dv}{dx} dx \overset{5\ 5}{=} \int_c^b u \frac{dv}{dx} dx + \int_a^c u \frac{dv}{dx} dx \overset{5}{\underset{5}{=}} uv \Big|_{c^+}^b - \int_{c^+}^b v \frac{du}{dx} dx + uv\Big|_a^{c^-} - \int_a^{c^-} v \frac{du}{dv} dx =$$

$$\overset{5}{\underset{5}{=}} uv \Big|_a^b + uv \Big|_{c^+}^{c^-} - \int_a^b v \frac{du}{dx} dx. \qquad (1)$$

10 5 5 b). if u, v are continuous accrose $c \Rightarrow$ $uv \Big|_{c^+}^{c^-} = 0.$ $\Rightarrow$ (1) reduces to the int.-by-part for.

3.4.2.

$f(x)$, $\frac{df}{dx}$ - piecewise smooth. $\overset{?}{\rightarrow}$ F.S.(f(x)) can be diff. term by term if F.S.(f(x)) is continuous.

$$f(x) \sim a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin \frac{n\pi x}{L} \qquad (f\text{-cont.}) \quad 5.$$

$$f'(x) \sim A_0 + \sum_{n=1}^{\infty} A_n \cos \frac{n\pi x}{L} + B_n \sin \frac{n\pi x}{L} \qquad (f'\text{-cont.}).$$

- $A_0 = \frac{1}{2L} \int_{-L}^{L} f'(x) dx = \frac{1}{2L} (f(L) - f(-L)) \underset{\uparrow}{=} 0$ 2

F.S.(f) continuous.

- $A_n = \frac{1}{L} \int_{-L}^{L} f'(x) \cos \frac{n\pi x}{L} dx = \frac{1}{L}\left(f(x) \cos \frac{n\pi x}{L} \Big|_{-L}^{L} + \frac{n\pi}{L} \int_{-L}^{L} f(x) \sin \frac{n\pi x}{L} dx \right)$ 5

$= \frac{1}{L} \left(f(L) \cos n\pi - f(-L) \cos n\pi \right) + \frac{n\pi}{L} b_n \underset{\uparrow}{=} \frac{n\pi}{L} b_n. \Rightarrow A_n = \frac{n\pi}{L} b_n, n \ge 1.$

from F.S.(f) cont. $\Rightarrow f(L) = f(-L)$.

- Analoguously $B_n = -\frac{n\pi}{L} a_n$, $n \ge 1$. 5

$\Rightarrow$ F.S(f(x)) can be differentiated term by term. 3

3.4.3.