3.4.3: f is continuous (except for a jump discont. at $x = x_0$, $f(x_0^-) = \alpha$, $f(x_0^+) = \beta$),
$\frac{df}{dx}$ - piecewise smooth.

a) F. Sines. S. $\left(\frac{df}{dx}\right)$ in term of F. C. S. $(f(x))$

① We study only the case when $x_0 \in (-L, L)$, others is trivial (no change!).

$$f(x) \sim b_0 + \sum_{n=1}^{\infty} b_n \cos \frac{\pi n x}{L}$$

$$f'(x) \sim \sum_{n=1}^{\infty} B_n \sin \frac{n\pi x}{L}.$$

$$B_n = \frac{1}{L}\int_{-L}^{L} \frac{df}{dx} \cdot \sin \frac{n\pi x}{L} dx \overset{②}{=} \frac{1}{L}\int_{x_0}^{L} \frac{df}{dx} \sin \frac{n\pi x}{L} dx + \frac{1}{L}\int_{-L}^{x_0} \frac{df}{dx} \sin \frac{n\pi x}{\alpha L} dx, =$$

$$\overset{④}{=} \frac{1}{L}\left(f(x) \sin \frac{n\pi x}{L} \Big|_{x_0^+}^{L} - \frac{n\pi}{L}\int_{x_0}^{L} f(x) \cos \frac{n\pi x}{L} dx \right) +$$

$$+ \frac{1}{L}\left(f(x) \sin \frac{n\pi x}{L} \Big|_{-L}^{x_0^-} - \frac{n\pi}{L}\int_{-L}^{x_0} f(x) \cos \frac{n\pi x}{L} dx \right) =$$

$$\overset{④}{=} \frac{1}{L} \cdot \left(\sin \frac{n\pi x_0}{L}\right)(\alpha - \beta) - \frac{n\pi}{L} \cdot b_n.$$

b) Analoguously

$$A_n = \frac{1}{L} \cdot \cos \frac{n\pi x_0}{L} \cdot (\alpha - \beta) + \frac{n\pi}{L} a_n.$$