Second Midterm Practice Exam, Math 412

Name: Key

SHOW ALL WORK!

Problem 1. Solve the PDE

$$\partial_{tt}u - 9\partial_{xx}u = 0, \qquad -\infty < x < \infty,\ t \geq 0,$$
$$u(x,0) = \sin x + \cos\tfrac{x}{3}, \quad \partial_t u(x,0) = 3\cos x - \sin\tfrac{x}{3}, \qquad -\infty < x < \infty.$$

$c^2 = 9 \Rightarrow c = 3$; $f = \sin x + \cos\frac{x}{3}$

$$u(x,t) = \frac{1}{2}f(x-3t) + \frac{1}{2}f(x+3t) + \frac{1}{6}\int_{x-3t}^{x+3t} 3\cos y - \sin\frac{y}{3}\,dy$$

$$u(x,t) = \frac{1}{2}\sin(x-3t) + \frac{1}{2}\sin(x+3t) + \frac{1}{2}\cos\left(\frac{x-3t}{3}\right) + \frac{1}{2}\left(\cos\frac{x+3t}{3}\right)$$

$$+ \frac{1}{2}\left(\sin(x+3t) - \sin(x-3t)\right) + \frac{1}{2}\left(\cos\frac{x+3t}{3} - \cos\frac{x-3t}{3}\right)$$

$$\boxed{u(x,t) = \sin(x+3t) + \cos\frac{x+3t}{3}}$$

Check ① $u(x,t) = F(x+3t) \rightarrow$ solves $u_{tt} = 3^2 u_{xx}$ ✓

② $u(x,0) = \sin x + \cos\frac{x}{3}$ ✓

③ $u_t(x,0) = 3\cos x - \cos\frac{x}{3}$ ✓