

**Problem 5.** Consider the conservation equation

$$\partial_t u + u \partial_x u = 0, \quad x \in (-\infty, \infty), \quad t > 0$$

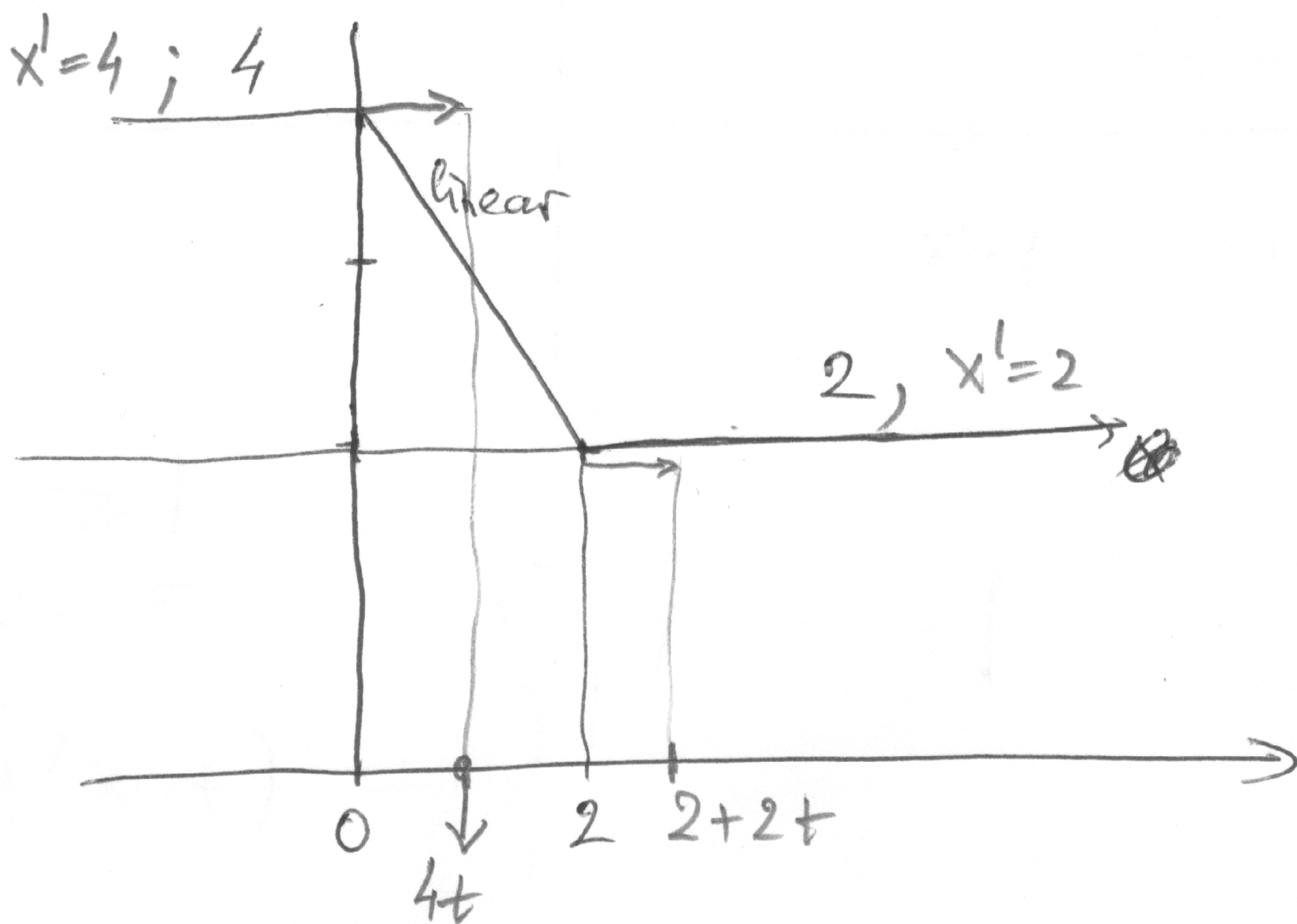
with the initial condition

$$\begin{aligned} u(x, 0) &= 4, & \text{if } x < 0, \\ u(x, 0) &= 4 - x, & \text{if } 0 < x < 2, \\ u(x, 0) &= 2, & \text{if } x > 2. \end{aligned}$$

$$u_t + \left(\frac{u^2}{2}\right)_x = 0$$


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(i) Solve this problem for  $0 \leq t \leq 1$ .



$$u(x, t) = \begin{cases} 4 & \text{if } x < 4t \\ \text{linear} & \text{if } 4t \leq x \leq 2+2t \\ 2 & \text{if } x > 2+2t \end{cases}$$

for  $t < 1$   $u = \text{p.w. linear continuous}$

When  $\underline{t = 1}$  the "linear" part goes away (jump)