

Extra homeworks problems, Math 412 – Popov

Name: .....

**SHOW ALL WORK!**

**Problem 1.** Consider the conservation equation

$$\partial_t u + u \partial_x u = 0, \quad x \in (-\infty, \infty), \quad t > 0$$

with the initial condition

$$\begin{aligned} u(x, 0) &= 4, & \text{if } x < 0, \\ u(x, 0) &= 4 - x, & \text{if } 0 < x < 3, \\ u(x, 0) &= 1, & \text{if } 3 < x < 4, \\ u(x, 0) &= 12, & \text{if } x > 4. \end{aligned}$$

Solve this problem for  $t > 0$ .

**Problem 2.** Consider the conservation equation

$$u_t + f(u)_x = 0, \quad x \in (-\infty, \infty), \quad t > 0$$

with the flux  $f$  given by

$$f(u) = \frac{1}{4}u(1 - u), \quad \text{if } u < 0.5,$$

$$f(u) = \frac{1}{2}u(u - 1) + \frac{3}{16}, \quad \text{if } u > 0.5.$$

and the initial condition

$$u(x, 0) = 0, \quad \text{if } x < 0.25,$$

$$u(x, 0) = 1, \quad \text{if } x > 0.25.$$

Solve this problem for  $0 < t \leq 1$ .

**Problem 3.** Consider the conservation equation

$$u_t + f(u)_x = 0, \quad x \in (-\infty, \infty), \quad t > 0$$

with the flux  $f$  given by  $f(u) = \frac{u^2}{u^2 + (1-u)^2}$  and the initial condition

$$u(x, 0) = 1, \quad \text{if } x < 0,$$
$$u(x, 0) = 0, \quad \text{if } x > 0.$$

Solve this problem for  $t > 0$ .