Math 147 Exam 3 Review

1a) $\lim_{x \to 0} \frac{\cos x - 1}{x^2}$ $\left(\frac{0}{0} \text{ form}\right)$

$= \lim_{x \to 0} \frac{-\sin x}{2x}$ $\left(\frac{0}{0} \text{ form again}\right)$

$= \lim_{x \to 0} \frac{-\cos x}{2}$ (plug in 0)

$= \boxed{-\frac{1}{2}}$

1b) $\lim_{x \to 0^+} 2x \cot x$ $(0 \cdot \infty \text{ form})$

$= \lim_{x \to 0^+} \frac{2x}{\frac{1}{\cot x}}$

$= \lim_{x \to 0^+} \frac{2x}{\tan x}$ $\left(\frac{0}{0} \text{ form}\right)$

$= \lim_{x \to 0^+} \frac{2}{\sec^2 x}$

$= \boxed{2}$

1c) $\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{4x}$ $\quad (1^\infty \text{ form})$

$= \lim_{x\to\infty} e^{\ln\left(1+\frac{2}{x}\right)^{4x}}$

$= e^{\lim_{x\to\infty} 4x\ln\left(1+\frac{2}{x}\right)}$

$= \boxed{e^8}$

$\lim_{x\to\infty} 4x\ln\left(1+\frac{2}{x}\right)$ $\quad (\infty\cdot 0 \text{ form})$

$= \lim_{x\to\infty} \dfrac{\ln\left(1+\frac{2}{x}\right)}{\frac{1}{4x}}$ $\quad \left(\frac{0}{0} \text{ form}\right)$

$= \lim_{x\to\infty} \dfrac{\frac{1}{1+2/x}\cdot\left(-\frac{2}{x^2}\right)}{\frac{-1}{4x^2}}$

$= \lim_{x\to\infty} \dfrac{1}{1+\frac{2}{x}}\cdot\dfrac{+2}{x^2}\cdot\dfrac{+4x^2}{1}$

$= \lim_{x\to\infty} \dfrac{8}{1+\frac{2}{x}}$

$= 8$

1d) $\lim_{x \to 0^+} \left(\frac{1}{x}\right)^{\sqrt{x}}$ (∞^0 form)

$= \lim_{x \to 0^+} e^{\ln\left(\frac{1}{x}\right)^{\sqrt{x}}}$

$= \lim_{x \to 0^+} e^{\sqrt{x}\ln\left(\frac{1}{x}\right)}$

$= e^{\lim_{x \to 0^+} \sqrt{x}\ln\left(\frac{1}{x}\right)}$

$\lim_{x \to 0^+} \sqrt{x}\ln\left(\frac{1}{x}\right)$ ($0 \cdot \infty$ form)

$= \lim_{x \to 0^+} \frac{\ln\left(\frac{1}{x}\right)}{\frac{1}{\sqrt{x}}}$ ($\frac{\infty}{\infty}$ form)

$= \lim_{x \to 0^+} \frac{\frac{1}{\frac{1}{x}} \cdot -\frac{1}{x^2}}{\frac{-1}{2x^{3/2}}}$

$= \lim_{x \to 0^+} x \cdot \frac{-1}{x^2} \cdot \frac{-2x^{3/2}}{1}$

$= \lim_{x \to 0^+} 2x^{1/2}$

$= 0$

$= e^0$

$= \boxed{1}$

1e) $\lim_{x\to\infty} \frac{(\ln x)^2}{4x}$ ($\frac{\infty}{\infty}$ form)

$= \lim_{x\to\infty} \frac{2(\ln x)\cdot\frac{1}{x}}{4}$

$= \lim_{x\to\infty} \frac{1}{2}\cdot\frac{\ln x}{x}$ ($\frac{\infty}{\infty}$ form again)

$= \frac{1}{2}\lim_{x\to\infty} \frac{\frac{1}{x}}{1}$

$= \frac{1}{2}\lim_{x\to\infty} \frac{1}{x}$

$= \frac{1}{2}\cdot 0$

$= \boxed{0}$