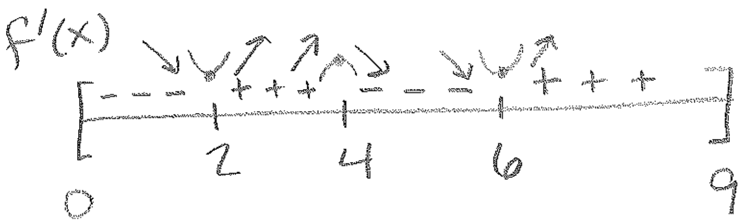
2) Let's assume that the graph of $f'(x)$ is defined on $[0, 9]$. We are told that the original function $f(x)$ is continuous, so let's assume it is continuous on $[0,9]$. To find critical values of $f(x)$, we look for values of x for which $f'(x) = 0$ or $f'(x)$ DNE. Looking at the graph of $f'(x)$, $f'(x) = 0$ at $x = 0$, $x = 2$, $x = 4$, and $x = 6$. These are the only critical values for $f(x)$ since $f'(x)$ exists for all $x \in (0,9)$. Now make a sign chart for $f'(x)$:



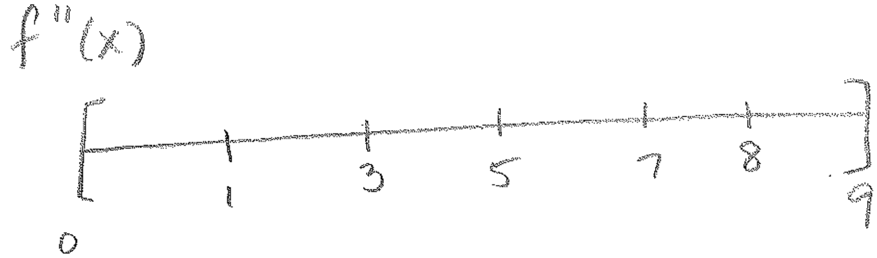
($f'(x) > 0$ where the graph of $f'(x)$ is above the x-axis, and v.v.)

So $f(x)$ (the original function) is increasing on $(2,4) \cup (6,9)$ and decreasing on $(0,2) \cup (4,6)$.

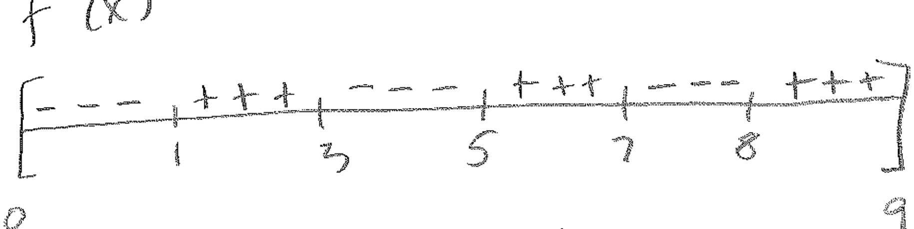
$f(x)$ has a local min at $x = 2$ and $x = 6$ and a local max at $x = 4$.

2) (Continued)

We must analyze $f''(x)$ to discuss concavity and inflection points. Note that $f''(x)$ is the derivative of $f'(x)$. Thus, $f''(x) = 0$ where the tangent line to $f'(x)$ is horizontal. Looking at the graph of $f'(x)$, there is a horizontal tangent at $x = 1, 3, 5, 7$, and 8. Now we make a sign chart for $f''(x)$:



Remember that the derivative of a function is positive where the function is increasing. This means $f''(x)$ is positive where $f'(x)$ is increasing. Also, $f''(x) < 0$ where $f'(x)$ is decreasing. Because of this, the sign chart becomes



Thus, $f(x)$ (the original function) is concave up on $(1,3) \cup (5,7) \cup (8,9)$ and concave down on $(0,1) \cup (3,5) \cup (7,8)$.

Also, $f(x)$ has inflection points at $x = 1, 3, 5, 7$, and 8.

3) a) $f(x) = x^3 - 5x^2 + 3$ on $[-1, 3]$ ← A <u>closed</u> interval

$f'(x) = 3x^2 - 10x = 0$

$x(3x - 10) = 0$

$x = 0$ or $x = \frac{10}{3} = 3\frac{1}{3}$ ← <u>not</u> in the interval!

End Points and Critical #'s

x	$f(x) = x^3 - 5x^2 + 3$
-1	$(-1)^3 - 5(-1)^2 + 3 = -3$
0	3 ← abs. max
3	$3^3 - 5(3)^2 + 3 = -15$ ← abs. min

The absolute (global) maximum value of $f(x)$ is 3 and the absolute (global) minimum value is -15.

3b) $f(x) = x \ln x$ on $[e^{-2}, 1]$ ← a <u>closed</u> interval!

$f'(x) = 1 \cdot \ln x + x \cdot \frac{1}{x}$

$0 = \ln x + 1$

$-1 = \ln x$

$e^{-1} = e^{\ln x}$

$x = e^{-1}$

Note: $e^{-2} \approx 0.1353$

$e^{-1} \approx 0.3679$ ← <u>Is</u> in the interval

End Points and Critical #'s

x	$f(x) = x \ln x$
e^{-2}	$e^{-2} \ln e^{-2} = -2e^{-2} \approx -0.27$ ← $\frac{-2}{e^2}$
e^{-1}	$e^{-1} \ln e^{-1} = -e^{-1} \approx -0.37$ ← $\frac{-1}{e}$
1	$1 \ln(1) = 0$

The absolute (global) max value of $f(x)$ is 0 and the absolute (global) min. value is $-\frac{1}{e}$.

4) $f(x) = x(6-x)^{\frac{1}{2}}$

Note: $6-x$ must be 0 or larger for the square root to produce a real number, so

$$6-x \geq 0$$
$$-x \geq -6$$
$$x \leq 6$$

Domain of $f(x)$ is $(-\infty, 6]$

$$f'(x) = 1\cdot(6-x)^{\frac{1}{2}} + x\cdot\tfrac{1}{2}(6-x)^{-1/2}(-1)$$

$$= (6-x)^{\frac{1}{2}} - \frac{x}{2(6-x)^{1/2}} = 0$$

$$\frac{(6-x)^{\frac{1}{2}}}{1} = \frac{x}{2(6-x)^{1/2}}$$

$$x = 2(6-x)$$

$$x = 12 - 2x$$

$$3x = 12$$

$\boxed{x = 4}$ ← critical number

Note that $f'(x)$ DNE at $x=6$ (this would make a denominator 0). $x=6$ is in the domain of the original function, so one could argue that it is a critical number for $f(x)$. However, $x=6$ is the right endpoint of the domain, and local extrema cannot occur at endpoints.

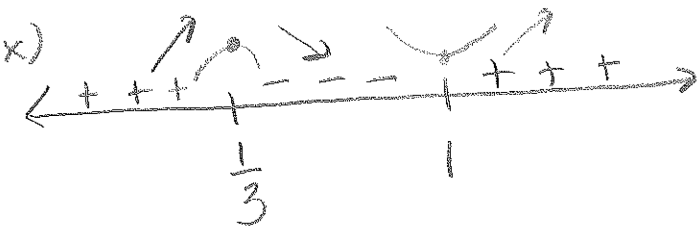
5a) $f(x) = x^3 - 2x^2 + x$

$f'(x) = 3x^2 - 4x + 1 = 0$

$(3x-1)(x-1) = 0$

$x = \frac{1}{3}$ or $x = 1$ ← only critical numbers

$f'(x)$



Test #	$f'(x)$
0	1
$\frac{2}{3}$	$-\frac{1}{3}$
2	5

$f(\frac{1}{3}) = (\frac{1}{3})^3 - 2(\frac{1}{3})^2 + \frac{1}{3} = \frac{4}{27}$

$f(1) = 1^3 - 2(1)^2 + 1 = 0$

$f(x)$ is increasing on $(-\infty, \frac{1}{3}) \cup (1, \infty)$ and decreasing on $(\frac{1}{3}, 1)$. $f(x)$ has a local maximum at $(\frac{1}{3}, \frac{4}{27})$ and a local minimum at $(1, 0)$.

5b) $f(x) = x^2 e^{2x}$ ← $f(-1) = (-1)^2 e^{2(-1)} = e^{-2}$

$f'(x) = 2xe^{2x} + x^2 e^{2x}(2) = 0$

$f(0) = 0^2 \cdot e^0 = 0$

$2xe^{2x}(1+x) = 0$

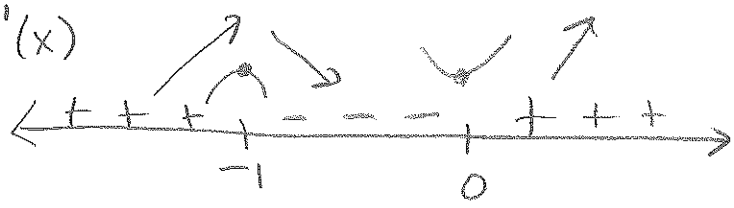
$2x = 0$ or $\underbrace{e^{2x} = 0}_{\text{no soln.}}$ or $1 + x = 0$

$x = 0$ $\qquad$ $x = -1$

only critical #'s
($f'(x)$ exists for all x)

5b) (continued)

$f'(x)$



Test #	$f'(x)$
-2	$(-)(+)(-) > 0$
$-\frac{1}{2}$	$(-)(+)(+) < 0$
1	$(+)(+)(+) > 0$

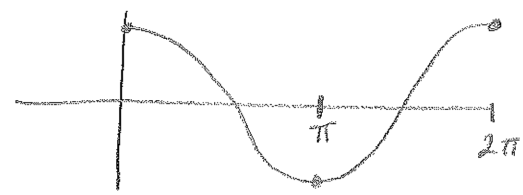
$f(x)$ is increasing on $(-\infty, -1) \cup (0, \infty)$ and decreasing on $(-1, 0)$. $f(x)$ has a local max at the point $(-1, e^{-2})$ and a local min at the point $(0,0)$.

5c) $f(x) = \sin x + x$ on $[0, 2\pi]$

$f'(x) = \cos x + 1 = 0$

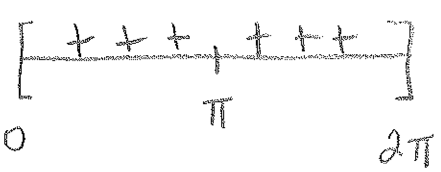
$\underbrace{\cos x = -1}$

True when $x = \pi$



← only critical # in $[0, 2\pi]$.

$f'(x)$



Test #	$f'(x)$
$\frac{\pi}{2}$	$1 > 0$
$\frac{3\pi}{2}$	$1 > 0$

$f(x)$ is increasing on $(0, \pi) \cup (\pi, 2\pi)$. $f(x)$ has no local extrema on $[0, 2\pi]$