6) $f(x) = x^4 + 8x^3 + 13$ ← Polynomial

$f'(x) = 4x^3 + 24x^2$ (Exists for all x)

$f''(x) = 12x^2 + 48x = 0$

$12x(x+4) = 0$

$x = 0$ or $x = -4$

$f''(x)$: $+++$ | -4 | $---$ | 0 | $+++$

Test #	$f''(x) = 12x(x+4)$
-5	$12(-5)(-5+4) > 0$
-2	$12(-2)(-2+4) < 0$
1	$12(1)(1+4) > 0$

f(x) is concave up on $(-\infty, -4) \cup (0, \infty)$ and concave down on $(-4, 0)$. f(x) has inflection points at $x = -4$ and $x = 0$ (coordinates: $(-4, -243)$ and $(0, 13)$.)

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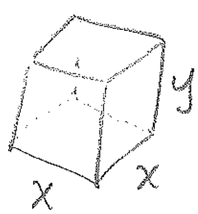
7) 

Volume $= x^2 y = 32 \text{ in}^3 \Rightarrow y = \frac{32}{x^2}$

$\underbrace{\text{Surface Area}} = x^2 + 4xy$ $\quad x \geq 0 \quad y \geq 0$

$f(x) = x^2 + 4x\left(\frac{32}{x^2}\right)$ $\quad \frac{32}{x^2} \geq 0$ ← $x \neq 0$

$$\boxed{\text{Minimize } f(x) = x^2 + \frac{128}{x} \text{ on } (0, \infty)}$$

$f'(x) = 2x - \frac{128}{x^2} = 0$

$\frac{2x}{1} = \frac{128}{x^2}$

$128 = 2x^3$ → $64 = x^3$

$4 = x$

only critical # in $(0, \infty)$.

→

7) (Continued)

We must verify that $x=4$ produces the global minimum for $f(x)=x^2+\frac{128}{x}$ on $(0,\infty)$. You may do this with a sign chart for $f'(x)$, or you may use the second derivative test. (Since the domain of this model is not a closed interval, we cannot simply plug in the end points and critical numbers to the original function)

$$f'(x)=2x-128x^{-2}$$

The 2nd derivative test.

$$f''(x)=2+256x^{-3}=2+\frac{256}{x^3}$$

$$f''(4)=2+\frac{256}{4^3}>0 \Rightarrow + +$$

← min

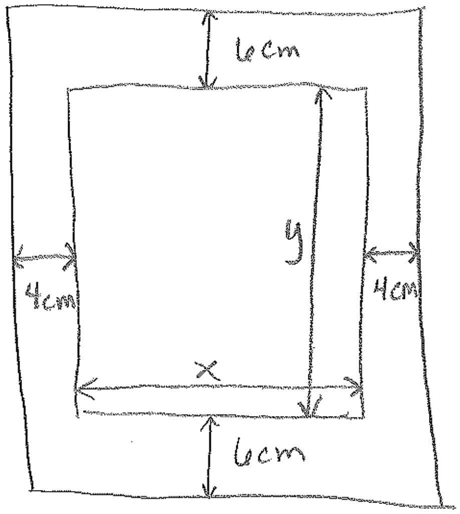
Thus, $f(x)$ has a local min at $x=4$. Since $x=4$ is the only critical number in $(0,\infty)$, $x=4$ is the location of the global minimum for $f(x)$ on $(0,\infty)$.

If $y=\frac{32}{x^2}$ and $x=4$,

$$y=\frac{32}{4^2}=\frac{32}{16}=2$$

To minimize the amount of material used, the box should be 4in × 4in × 2in.

8)



Printed Area $= xy = 384\text{cm}^2 \Rightarrow y = \dfrac{384}{x}$

$\underbrace{\text{Poster Area}} = (\text{length})(\text{width})$

$$A(x) = (x+8)(y+12)$$

$$A(x) = (x+8)\left(\frac{384}{x} + 12\right)$$

$$A(x) = 384 + 12x + \frac{3072}{x} + 96$$

$$A(x) = 12x + \frac{3072}{x} + 480$$

$x \geq 0 \quad y \geq 0$

$y \geq 0 \downarrow \dfrac{384}{x} \geq 0 \leftarrow x \neq 0.$

Goal:

$$\boxed{\text{Minimize } A(x) = 12x + \frac{3072}{x} + 480 \text{ on } (0,\infty)}$$

$$A'(x) = 12 - \frac{3072}{x^2} = 0$$

$$\frac{12}{1} = \frac{3072}{x^2}$$

$$12x^2 = 3072$$

☆fix!

$$x^2 = 256$$

$x = \pm 16 \leftarrow$ take $x = 16$

Second Deriv. Test

$$A'(x) = 12 - 3072x^{-2}$$

$$A''(x) = 6144x^{-3} = \frac{6144}{x^3}$$

$$A''(16) = \frac{6144}{16^3} > 0 \Rightarrow ++$$

$\leftarrow$ min

$\Rightarrow$ Global min at $x=16$ (since only critical # in $(0,\infty)$).

$x = 16,\ y = \dfrac{384}{16} = 24$

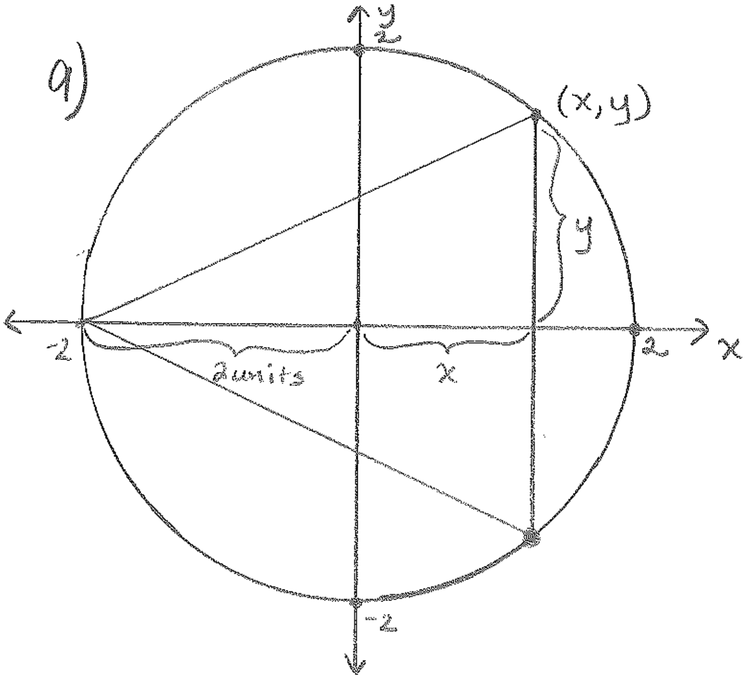
$\boxed{\text{The poster with smallest area has dimensions 24cm} \times \text{36cm}}$

$x+8$ by $y+12$

$16+8$ by $24+12$

24cm by 36cm

9)



Eqn of a circle

$$x^2+y^2=r^2$$

$$x^2+y^2=2^2$$

$$y^2=4-x^2$$

$$y=\sqrt{4-x^2} \leftarrow \text{top half}$$

or

$$y=-\sqrt{4-x^2} \leftarrow \text{bottom half}$$

$$\text{Area} = \tfrac{1}{2}(\text{base})(\text{height})$$

$$A(x)=\tfrac{1}{2}(2y)(x+2)$$

$$A(x)=y(x+2)=\sqrt{4-x^2}\cdot(x+2)$$

Goal: $\boxed{\text{Maximize } A(x)=(x+2)\sqrt{4-x^2} \text{ on } [-2,2]}$.

$\nwarrow$ A closed interval!

$$A'(x)=1\cdot\sqrt{4-x^2}+(x+2)\cdot\tfrac{1}{2}(4-x^2)^{-\frac{1}{2}}(-2x)$$

$$A'(x)=\sqrt{4-x^2}-\frac{\cancel{2}x(x+2)}{\cancel{2}\sqrt{4-x^2}}=0$$

End Pts + Critical #'s

x	$A(x)=(x+2)\sqrt{4-x^2}$
-2	0
$\boxed{1}$	$\boxed{3\sqrt{3}}$ $\leftarrow$ max
2	0

$$\frac{\sqrt{4-x^2}}{1}=\frac{x^2+2x}{\sqrt{4-x^2}}$$

$$x^2+2x=4-x^2$$

$$2x^2+2x-4=0$$

$$2(x^2+x-2)=0$$

$$2(x+2)(x-1)=0$$

$$x=-2 \text{ or } x=1$$

Conclusion

$x=1$ so $y=\sqrt{4-1^2}=\sqrt{3}$

base $=2y=2\sqrt{3}$

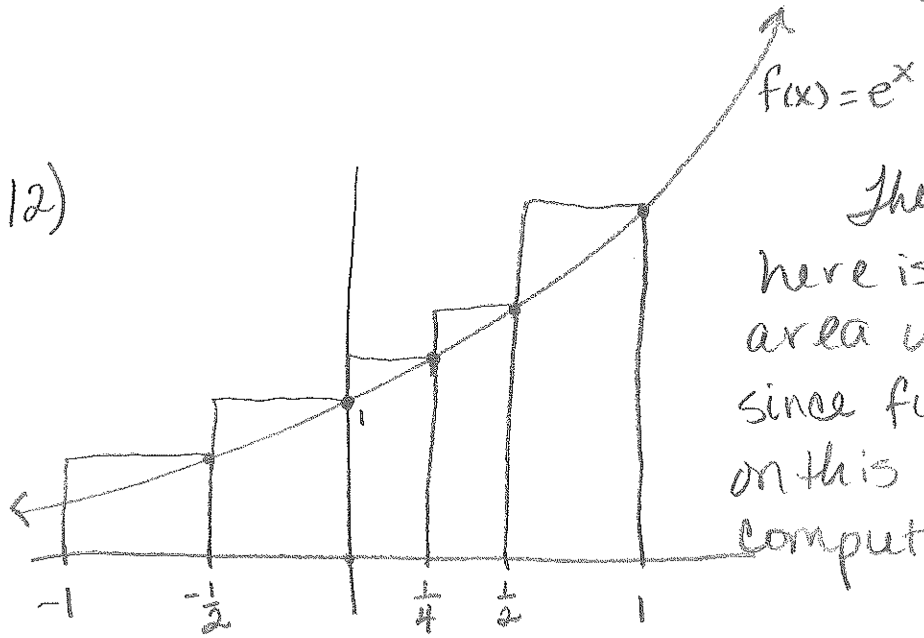
height $=x+2=3$

The isosceles triangle with max. area has a base $=2\sqrt{3}$ units and a height of 3 units.

10) $\sum_{i=1}^{5} i^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2 = \boxed{55}$

11) $1 + \frac{1}{e} + \frac{1}{e^2} + \frac{1}{e^3} + \frac{1}{e^4} + \frac{1}{e^5} = \boxed{\sum_{k=0}^{5} \frac{1}{e^k}}$ or $\sum_{k=0}^{5} e^{-k}$

12)



The Riemann sum shown here is an overestimate of the area under $f(x) = e^x$ on $[-1, 1]$ since $f(x)$ is (monotonic) increasing on this interval and we are computing a right Riemann sum.

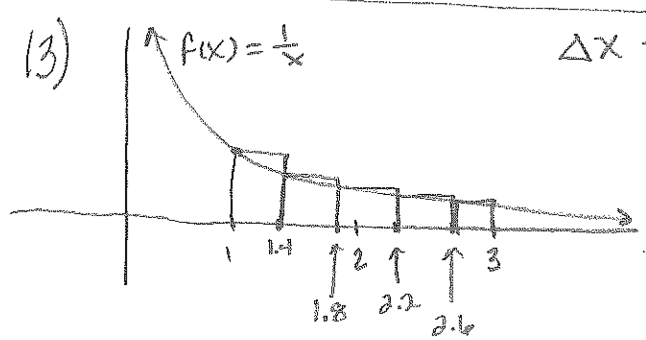
$$\text{Riemann Sum} = \tfrac{1}{2}\cdot f(-\tfrac{1}{2}) + \tfrac{1}{2} f(0) + \tfrac{1}{4} f(\tfrac{1}{4}) + \tfrac{1}{4} f(\tfrac{1}{2}) + \tfrac{1}{2} f(1)$$

$$= \tfrac{1}{2} e^{-1/2} + \tfrac{1}{2}\cdot 1 + \tfrac{1}{4} e^{1/4} + \tfrac{1}{4} e^{1/2} + \tfrac{1}{2} e$$

$$\approx \boxed{2.8956}$$

---

13)



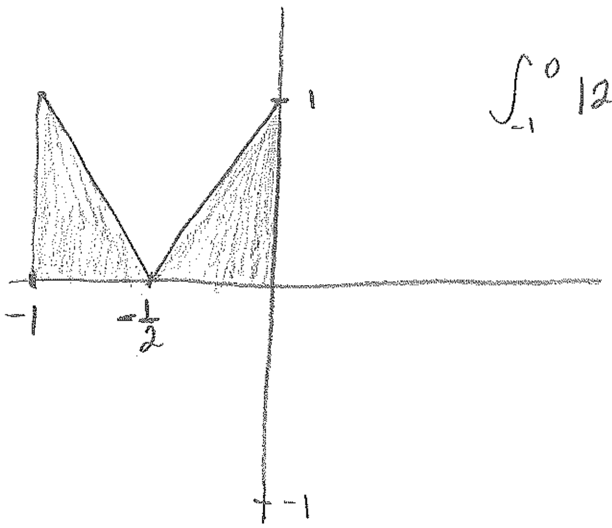
$\Delta x = \frac{b-a}{n} = \frac{3-1}{5} = \frac{2}{5}$ = width of each rectangle
$= 0.4$

$L_5 = 0.4\left(f(1) + f(1.4) + f(1.8) + f(2.2) + f(2.6)\right)$

$L_5 = 0.4\left(1 + \frac{1}{1.4} + \frac{1}{1.8} + \frac{1}{2.2} + \frac{1}{2.6}\right)$

$\boxed{L_5 \approx 1.2436}$

14) $\int_{-1}^{0} |2x+1|\,dx$ ← The area under $f(x)=|2x+1|$ on $[-1, 0]$.



$$\int_{-1}^{0} |2x+1|\,dx = \overset{\frac{1}{2}\cdot b\cdot h}{\frac{1}{2}\cdot\frac{1}{2}\cdot 1} + \overset{\frac{1}{2}\cdot b\cdot h}{\frac{1}{2}\cdot\frac{1}{2}\cdot 1}$$

$$= \frac{1}{4}+\frac{1}{4}$$

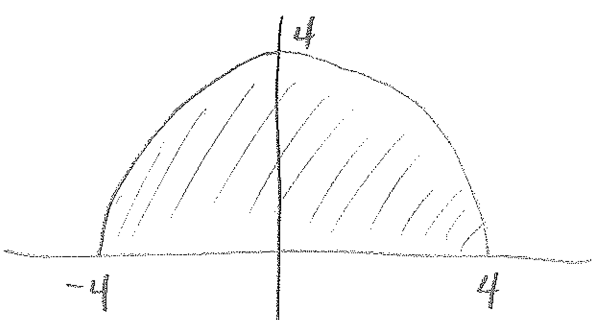
$$= \boxed{\frac{1}{2}}$$

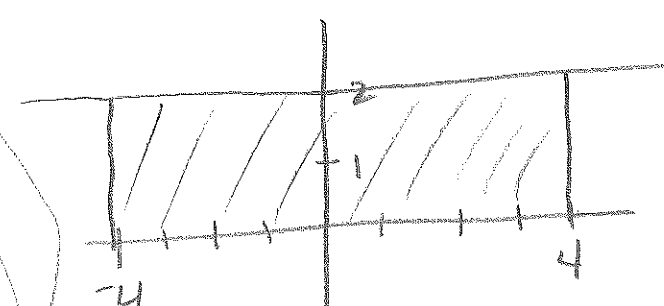
15) $\int_{-4}^{4}\left(\sqrt{16-x^2} - 2\right)dx = \int_{-4}^{4}\sqrt{16-x^2}\,dx - \int_{-4}^{4} 2\,dx$

$$= \boxed{8\pi - 16}$$

$y=\sqrt{16-x^2}$ is the top half of a circle of radius 4, centered at $(0,0)$

$y=2$ is a horiz. line





$\int_{-4}^{4}\sqrt{16-x^2}\,dx = \frac{1}{2}$ of Area of circle $= \frac{1}{2}(\pi\cdot 4^2) = 8\pi$

$\int_{-4}^{4} 2\,dx =$ area of rectangle $= 8\cdot 2 = 16$

16a) $\int_1^3 (x^2 + 2x)\,dx = \lim_{\|P\| \to 0} \sum_{k=1}^{n} (c_k^2 + 2c_k)\Delta x_k$

where $P = \{x_0, x_1, \ldots, x_n\}$ is a sequence of partitions of the interval $[1,3]$, $\|P\|$ is the norm of P (i.e., the width of the widest rectangle), Δx_k is the width of the k^{th} rectangle, and c_k is the randomly chosen x-value in the k^{th} subinterval used to determine the height of the k^{th} rectangle.

16b) $\int_{\frac{\pi}{2}}^{\pi} (x\sin(x) + 2)\,dx = \lim_{\|P\| \to 0} \sum_{k=1}^{n} \left(c_k \sin(c_k) + 2\right)\Delta x_k$

where $P = \{x_0, x_1, \ldots, x_n\}$ is a sequence of partitions of the interval $[\frac{\pi}{2}, \pi]$, $\|P\|$ is the norm of P (i.e., the width of the widest rectangle), Δx_k is the width of the k^{th} rectangle, and c_k is the randomly chosen x-value in the k^{th} subinterval used to determine the height of the k^{th} rectangle.

17) $\lim_{\|P\| \to 0} \sum_{k=1}^{n} \sqrt{5c_k + c_k^2}\, \Delta x_k = \boxed{\int_1^4 \sqrt{5x + x^2}\, dx}$

18) $\int_{-3}^{5} f(x)\,dx - \int_{-3}^{0} f(x)\,dx + \int_{5}^{6} f(x)\,dx$

$= \int_{-3}^{0} f(x)\,dx + \int_{0}^{5} f(x)\,dx - \int_{-3}^{0} f(x)\,dx + \int_{5}^{6} f(x)\,dx$

cancel

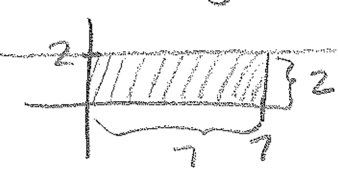
$= \int_{0}^{5} f(x)\,dx + \int_{5}^{6} f(x)\,dx$

$= \boxed{\int_0^6 f(x)\,dx}$

19) Given: $\int_0^5 f(x)\,dx = 10$ and $\int_5^7 f(x)\,dx = 3$

$\int_0^7 (4f(x) + 2)\,dx = \int_0^7 4f(x)\,dx + \underbrace{\int_0^7 2\,dx}$

$= 4\int_0^7 f(x)\,dx +$ area of a rectangle

$= 4\left(\int_0^5 f(x)\,dx + \int_5^7 f(x)\,dx\right) +$ 

$= 4(10 + 3) + 14$

$= \boxed{66}$