Math 147 Exam 3 Review - Fall 2012

1. Find the limits of each of the following:

   a) $\lim_{x \to 0} \frac{\cos x - 1}{x^2}$

   b) $\lim_{x \to 0^+} 2x \cot x$

   c) $\lim_{x \to \infty} \left(1 + \frac{2}{x}\right)^{4x}$

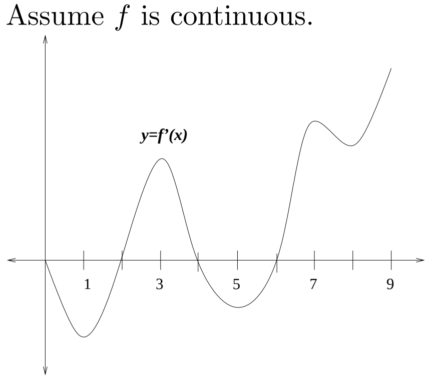
   d.) $\lim_{x \to 0^+} \left(\frac{1}{x}\right)^{\sqrt{x}}$

   e.) $\lim_{x \to \infty} \frac{(\ln x)^2}{4x}$

2. Using the graph of $f'(x)$ below, determine

   - all critical values of $f(x)$.
   - intervals where $f(x)$ is increasing and decreasing.
   - local extrema of $f(x)$.
   - intervals where $f(x)$ is concave up and concave down.
   - the x-coordinates of the inflection points of $f(x)$.

   Assume f is continuous.



3. Find the absolute maximum and minimum of the given function on the given interval.

   a) $x^3 - 5x^2 + 3$ on $[-1, 3]$

   b) $x \ln x$ on $[e^{-2}, 1]$

4. Find the critical numbers for $f(x) = x\sqrt{6 - x}$

5. Find the intervals where the given function is increasing and decreasing and identify all local extrema:

   a) $f(x) = x^3 - 2x^2 + x$

   b) $f(x) = x^2 e^{2x}$

   c) $f(x) = \sin x + x$, $0 \leq x \leq 2\pi$

6. Determine where the graph of $f(x) = x^4 + 8x^3 + 13$ is concave up and concave down and find the inflection points.

7. A cardboard box holding 32 cubic inches with a square base and open top is to be constructed. Find the dimensions of the box that minimize the amount of material used.

8. The top and bottom margins of a poster are each 6 cm and the side margins are each 4 cm. If the area of printed material on the poster is fixed at 384 square cm, find the dimensions of the poster with the smallest area.

9. Find the dimensions of the isosceles triangle of largest area that can be inscribed in a circle of radius 2.

10. Expand and find the sum: $\sum_{i=1}^{5} i^2$

11. Write $1 + \frac{1}{e} + \frac{1}{e^2} + \frac{1}{e^3} + \frac{1}{e^4} + \frac{1}{e^5}$ in summation notation.

12. Set up the Riemann sum to approximate the area under the graph of $f(x) = e^x$ over the interval $[-1, 1]$ by using the partition $P = \{-1, -\frac{1}{2}, 0, \frac{1}{4}, \frac{1}{2}, 1\}$ and choosing c_k to be the right endpoint of each interval. Construct the approximating rectangles. Does this over approximate or under approximate the true area? Support your answer.

13. Approximate $\int_1^3 \frac{1}{x}\, dx$ using 5 equally-spaced left rectangles.

14. Use a graph to interpret $\int_{-1}^{0} |2x + 1|\, dx$ in terms of an area.

15. Use an area formula from geometry to find the value of $\int_{-4}^{4} \left(\sqrt{16 - x^2} - 2\right)\, dx$ by interpreting it as the (signed) area under the graph.

16. Express the following definite integrals as a limit of infinte sums.

    (a) $\int_1^3 (x^2 + 2x)\, dx$

    (b) $\int_{\pi/2}^{\pi} (x \sin(x) + 2)\, dx$

17. Express $\lim_{||P|| \to 0} \sum_{k=1}^{n} \sqrt{5c_k + c_k^2}\ \Delta x_k$, where $P = [x_0, x_1, x_2, x_3, ..., x_n]$, $n = 1, 2, 3, ...,$ is a sequence of partitions of [1, 4] into n subintervals, $c_k \in [x_{k-1}, x_k]$, and $\Delta x_k = x_k - x_{k-1}$.

18. Express the following as a single integral, if possible.
$\int_{-3}^{5} f(x)\,dx - \int_{-3}^{0} f(x)\,dx + \int_{5}^{6} f(x)\,dx$

19. Given $\int_{0}^{5} f(x)\,dx = 10$ and $\int_{5}^{7} f(x)\,dx = 3$, find $\int_{0}^{7} (4f(x) + 2)\,dx$

**Note: There are no problems from 2.1, 2.2, or 5.6 on this review. To study for these sections refer to recitation activities, quizzes, and suggested homework.