Math 166 - Week in Review #10

Section 9.1 - Markov Chains

- Markov Process (or Markov Chain) - a special class of stochastic processes in which the probabilities associated with the outcomes at any stage of the experiment depend *only* on the outcomes of the preceding stage.

- The outcome at any stage of the experiment in a Markov process is called the *state* of the experiment.

- Transition Matrix - A transition matrix associated with a Markov chain with n states is an $n \times n$ matrix T with entries $a_{ij} = P(\text{moving to state } i | \text{ currently in state } j)$ such that

  1. $a_{ij} \geq 0$ for all i and j.
  2. The sum of the entries in each column of T is 1.

- Any matrix satisfying the two properties above is called a *stochastic* matrix.

- If T is the transition matrix associated with a Markov process, then the probability distribution of the system after m observations (or steps) is given by
$$X_m = T^m X_0$$

Section 9.2 - Regular Markov Chains

- The goal of this section is to investigate long-term trends of certain Markov chains.

- Regular Markov Chain - A stochastic matrix T is a **regular Markov chain** if the sequence $T, T^2, T^3, \ldots$ approaches a steady state matrix in which all entries are *positive* (i.e., strictly greater than 0).

- It can be shown that a stochastic matrix T is regular if and only if *some* power of T has entries that are all positive.

- **Finding the Steady-State Distribution Vector** - Let T be a regular stochastic matrix. Then the steady-state distribution vector X may be found by solving the vector equation
$$TX = X$$
together with the condition that the sum of the elements of the vector X must equal 1 (i.e., $x_1 + x_2 + \cdots + x_n = 1$).

1. Acme Taxi services the Bryan/College Station area. At 8am on one particular day, 38% of all of their taxi cabs were in Bryan and the rest were in College Station. According to Acme's records, 35% of all passengers picked up in Bryan ask to be driven to College Station, and 45% of all passengers picked up in College Station ask to be dropped off in Bryan.

   (a) Find the transition matrix and initial distribution vector for this system.

   (b) What is the distribution of the taxi cabs after each cab has transported 1 passenger? After transporting 8 passengers?

   (c) Is this a regular Markov chain? If yes, find its steady state vector and explain the meaning of its entries. If no, explain why.

2. The town of Gonzales has three restaurants that sell tacos. A study found that 75% of those who dine at Reyna's Taco Hut during a particular week will return to Reyna's in the following week, but 15% will dine at Matamoros Taco Hut in the following week, and the rest will go to Mr. Taco. Of those who dine at Matamoros Taco Hut in a particular week, 5% will then go to Reyna's Taco Hut, 15% will then go to Mr. Taco in the next week, and the rest will return to Matamoros. Of those who dine at Mr. Taco in a particular week, 10% will go to Reyna's, 20% will go to Matamoros Taco hut, and the rest will return to Mr. Taco in the next week. This week, 25% of those craving tacos went to Reyna's Taco Hut, 33% went to Matamoros Taco Hut, and the rest went to Mr. Taco.

   (a) Find the transition matrix and initial state vector for this Markov chain.

   (b) What percent of customers will visit the three restaurants during the next week? What about 5 weeks from now? 15 weeks from now?

   (c) Is this a regular Markov chain? If yes, find its steady state vector and explain the meaning of its entries. If no, explain why.

3. Which of the following are regular stochastic matrices? For each stochastic matrix that is regular, find the steady state distribution vector.

   (a) $\begin{bmatrix} 0.5 & 1 \\ 0.5 & 0 \end{bmatrix}$

   (b) $\begin{bmatrix} 0.4 & 0 & 0.1 \\ 0.1 & 1 & 0.3 \\ 0.5 & 0 & 0.6 \end{bmatrix}$

4. An insurance company found that 26% of the drivers in a particular community who are involved in an accident one year will also be involved in an accident in the following year. Only 13% of the drivers who are not involved in an accident in one year will be involved in an accident in the next year. In 2006, 6% of the drivers in this community were involved in an accident.

   (a) What is the probability that a driver in this community will be in an accident in 2007?

   (b) What is the probability that a driver in this community will be in an accident in 2010?