

**Math 142 - Exam 3 Review**

NOTE: Exam 3 covers sections 5.4-5.6, 6.1, 6.2, 6.4, 6.5, 7.1, and 7.2. This review is intended to highlight the material covered on Exam 3 but should not be used as your sole source of practice. Also refer to your instructor's lecture notes, previous week-in-reviews, suggested homework, supplemental homework, and the online homework as additional sources for review and exam preparation.

1. Find the area bounded between  $f(x) = x^3$  and  $g(x) = x$ .
  
  
  
  
  
  
  
  
  
  
2. The price-demand equation for a certain product is given by  $p = D(x) = 75 - 0.1x$  dollars per item, and the price-supply equation for this product is  $p = S(x) = 15e^{0.002x}$  dollars per item.
  - (a) Find the market equilibrium point. Round the equilibrium quantity to the nearest item, and round the equilibrium price to the nearest cent.
  
  
  
  
  
  
  
  
  
  
  - (b) Using your rounded answers in (a), find the producers' surplus at the equilibrium price level. What does this number represent?

3. Acme Media Company has a uniform annual demand for 21,600 DVDs. It costs \$0.50 to store a DVD for one year and \$294 to set up the machinery that produces their DVDs. How many times per year should Acme produce DVDs to minimize the total storage and setup costs?

4. Find the absolute extrema of  $f(x) = (5 - x)(x + 7)^2$  on

(a)  $[-9, -5]$

(b)  $(0, \infty)$

5. The U.S. Postal Service considers a package to be regular-sized if its length plus girth (distance around) does not exceed 84 inches.

(a) Find the dimensions of a rectangular box with square ends that satisfies this restriction and has maximum volume. What is the maximum volume?

(b) Find the dimensions (radius and height) of a cylindrical container that satisfies this restriction and has maximum volume. What is the maximum volume?

6. Find the equation of the curve that passes through the point  $(1, 9)$  if its slope is given by  $\frac{dy}{dx} = 8x^3 - 5x^{-1} + 4$  for all  $x \neq 0$ .

7. Compute each of the following by hand.

(a)  $\int (e^{10x} - x)(e^{10x} - 5x^2) dx$

(b)  $\int \frac{\ln x + 7}{x} dx$

(c)  $\int \left( 3\sqrt{t} + \frac{5}{\sqrt[3]{t^8}} + \frac{6}{t} - 1 \right) dt$

$$(d) \int_1^2 m^2(m^4 - 7m^{-6})dm$$

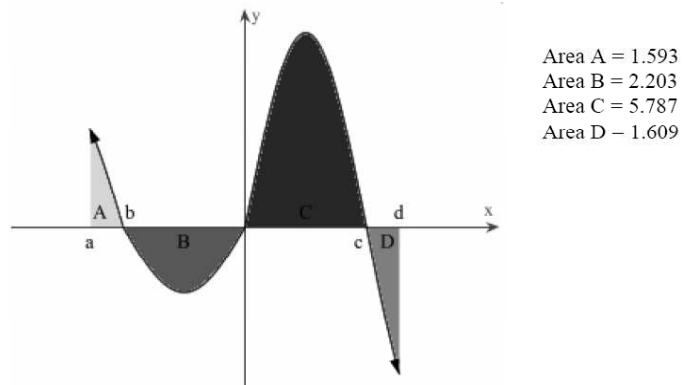
$$(e) \int_0^{\frac{1}{7}} \frac{e^{7x}}{5 - e^{7x}} dx$$

$$(f) \int z(z+4)^8 dz$$

8. The price-demand equation for a certain item is given by  $p = D(x) = -0.002(x + 100)^2 + 7000$  dollars per item, where  $x$  is the number of items that can be sold at a price of  $\$p$ . If the current price per item is  $\$4,580$ , find the consumers' surplus. What does this number represent?
9. Acme Furniture Company's marginal cost for its dinette sets is given by  $C'(x) = 3e^{0.01x} + \frac{375}{\sqrt{x}}$  dollars per set, where  $x$  is the number of dinette sets produced each month.
- (a) Find the change in total cost that results from going from a production level of 100 to 150 dinette sets per month.
- (b) If Acme's fixed cost for producing dinette sets is  $\$1,700$ , find a model for total cost.

10. Bob invested \$3,000 into an account paying 7.3% per year compounded continuously at the beginning of 2003. Find the average account balance during the second quarter of 2003.

11. Calculate the following definite integrals by referring to the graph of  $f(x)$  and indicated area below. (courtesy Jenn Whitfield)



(a)  $\int_b^0 f(x)dx$

(b)  $\int_a^c f(x)dx$

(c)  $\int_b^d f(x)dx$

(d)  $\int_d^c f(x)dx$

12. Approximate the area under  $f(x) = \sqrt{x-1} + 4$  on the interval  $[1, 7]$  using a midpoint sum with 3 rectangles of equal width. Include an appropriate sketch with your answer.
13. If  $f(x)$  in the previous problem represents the rate at which a swimming pool is filling with water and is given in gallons per minute, where  $x$  is the number of minutes since 5:30pm, what does the approximated area above represent?
14. Find the area between  $f(x) = 0.04x^2 - 1$  and  $g(x) = 0.2x + 1$  on the interval  $[-7, 7]$ .



15. Given that  $f(x) = \frac{3x^2 + 2}{x^2 - 25}$ ,  $f'(x) = \frac{-154x}{(x^2 - 25)^2}$ , and  $f''(x) = \frac{462x^2 + 3850}{(x^2 - 25)^3}$ ,

(a) Find the domain of  $f(x)$ .

(b) Find all intercepts of  $f(x)$ .

(c) Find all asymptotes of  $f(x)$ .

(d) Find all intervals where  $f(x)$  is increasing and all intervals where  $f(x)$  is decreasing.

(e) Find the coordinates of all local extrema.

(f) Find all intervals where  $f(x)$  is concave upward and all intervals where  $f(x)$  is concave downward.

(g) Find the coordinates of all inflection points.

(h) Sketch the graph of  $f(x)$ .

16. Suppose that  $x$  is a continuous random variable with associated probability density function

$$f(x) = \begin{cases} \frac{2}{x^3} & \text{if } x \geq 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the probability that  $x$  is less than 1.5.