Math 141 - Week in Review #10

Section 8.4 - Binomial Distribution

- Experiments with two outcomes ("success" and "failure") are called Bernoulli or binomial trials.
- Properties of a Binomial Experiment
  1. The number of trials in the experiment is fixed.
  2. There are two outcomes of the experiment: "success" and "failure."
  3. The probability of success in each trial is the same.
  4. The trials are independent of each other.
- NOTATION: n = number of trials, p = probability of success in a single trial, $q = 1 - p$ = probability of failure in a single trial, r = the number of successes wanted
- In binomial experiments, the binomial random variable X denotes the number of successes in the n trials of the experiment.

- The probability of obtaining exactly r successes in n binomial trials is given by $P(X=r)=C(n,r)p^r q^{n-r}$.
- Let X be a binomial random variable. Then $\mu = E(X) = np$, $\mathrm{Var}(X) = npq$, and $\sigma_x = \sqrt{npq}$, where n, p, and q are as defined above.

Section 8.5 - The Normal Distribution

- Properties of the Normal Curve

  1. The normal curve is completely determined by μ and σ. (σ determines the sharpness or flatness of the curve.)
  2. The curve has a peak at $x=\mu$.
  3. The curve is symmetric with respect to the vertical line $x=\mu$.

     area = .6827

  4. The curve always lies above the x-axis but approaches the x-axis as x extends indefinitely in either direction.
  5. The area under the curve and above the x-axis is 1.

  6. For any normal curve, 68.27% of the area under the curve lies within 1 standard deviation from the mean, 95.45% of the area lies within 2 standard deviations of the mean, and 99.73% of the area lies within 3 standard deviations of the mean.

- The *standard* normal random variable Z has mean 0 and standard deviation 1.

Section 8.6 - Applications of the Normal Distribution

- When approximating binomial probabilities by using the normal curve, first draw and shade a piece of a histogram corresponding to the probability you are being asked to find, and then use appropriate lower and upper bounds (adjust by 0.5) under the normal curve with $\mu = np$ and $\sigma = \sqrt{npq}$ to approximate the probability.

1. Which of the following experiments are binomial? Justify your answer.

   (a) Cast a fair die until a 3 lands up.

   Not binomial because there is no fixed # of trials.

   2 defectives

   (b) A box contains 20 clocks, 10% of which are defective. A sample of 5 clocks is selected one at a time without replacement and tested for quality control purposes.

   1) $n=5$

   2) 2 outcomes ("success" = getting a defective)

   3) Probabilities change with each trial → NOT binomial.

   (Trials are also not independent)

   (c) Draw 6 cards one at a time with replacement and record the suit of each card drawn.

   4 outcomes.

   1) $n=6$

   2) More than 2 outcomes, so not binomial.

   boys vs. girls

   (d) Analyze the composition of a 4-child family in which each child was born at a different time (no twins, triplets, etc.).

   1) $n=4$

   2) 2 outcomes – yes : pick boy to be success.

   3) $p=.5$ (same in each trial)

   4) Indep trials ✓

   Is a binomial experiment.

2. Consider the composition of an 8-child family in which each child was born at a different time.

(a) What is the probability that exactly 2 of the children are boys?

"exactly a certain # of successes"

$n=8$
success = boy
$p=.5$
Indep ✓

$P(X=2) = \text{binompdf}(8,.5,2) = \boxed{0.1094}$

BINOM option 1 R=2

(b) What is the probability that at most 2 of the children are boys?

0 or 1 or 2 boys

$n=8$
$p=.5$

$P(0 \le X \le 2) = \boxed{.1445} = \text{binomcdf}(8,.5,2)$

BINOM option 2 Lower R = 0, Upper R = 2

(c) What is the probability that at least 5 of the children are girls?

5 or more

$n=8$
"success" = girl
$p=.5$

$P(5 \le X \le 8) = \boxed{.3633} = 1 - \text{binomcdf}(8,.5,4) = \boxed{.3633}$

BINOM opt 2 Lower R = 5, Upper R = 8

(d) What is the probability that at least 3 but no more than 6 of the children are girls?

$n=8$
$p=.5$

$P(3 \le X \le 6) = \boxed{0.8203}$

BINOM option 2 Lower R = 3, upper R = 6

see next page also.

(e) How many of the children can you expect to be boys?

$E(X) = np = (8)(.5) = \boxed{4 \text{ boys}}$

(f) Find the variance and standard deviation of the number of boys.

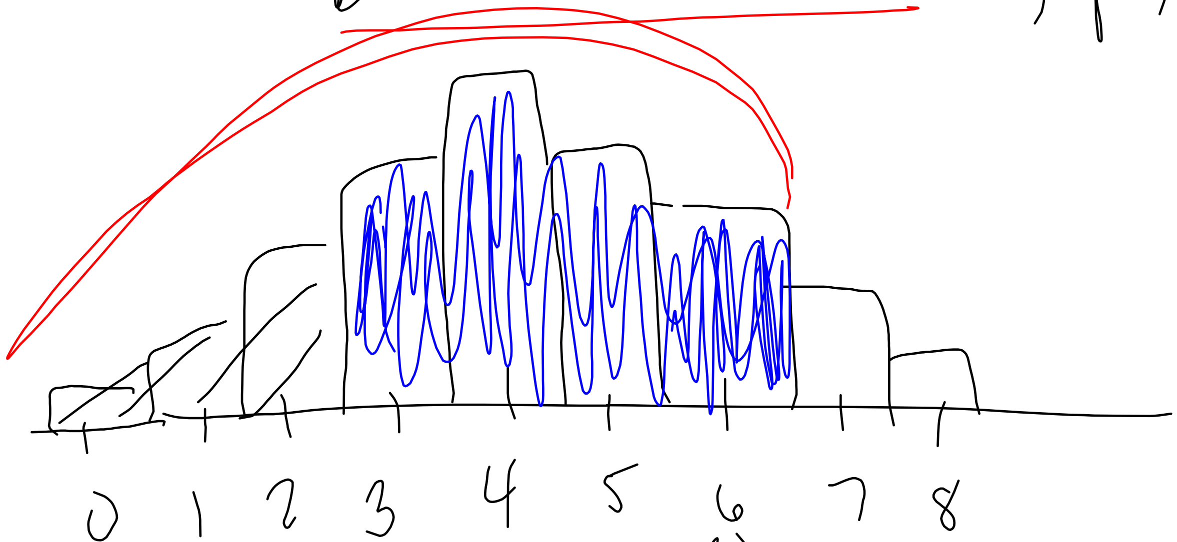
probability of failure $= 1-p$

$var(X) = npq = (8)(.5)(1-.5)$

$\boxed{Var(X) = 2}$

$\sigma_x = \sqrt{npq} = \sqrt{2} \approx \boxed{1.4142}$

$\text{binomcdf}(n, p, a) = P(X \leq a)$



answer to (d):

$\text{binomcdf}(8, .5, 6) - \text{binomcdf}(8, .5, 2)$

3. How many times must a person cast a die if the chances of obtaining at least 1 six are 70% or better?

Find n

"success" - getting a 6

$p = \frac{1}{6}$

Indep? ✓

Want $P(X \geq 1) = .7$ (or higher)

Try different values of n in the Binom program using option 2 with lower $R = 1$ and upper r equal to whatever n you are trying. You should find the the smallest number of trials needed to make $P(1 \leq X \leq n)$ (the probability of at least 1 six in n trials) is $\boxed{n = 7}$.

How to solve using binompdf:

P(atleast 1 six) = 1 − P(no 6's)

= 1 − binompdf($n, \frac{1}{6}, 0$)

Try different values of n until $1 - \text{binompdf}(n, \frac{1}{6}, 0) \geq .7$

$\boxed{n = 7}$

4. A health inspector has determined that 12% of all restaurants in a certain city are in violation of the health code. If 5 restaurants are selected at random for inspection, what is the probability that

(a) exactly 3 of the restaurants fail the inspection?

$n=5$

"success" = restaurant fails the inspection

$p=.12$

$P(X=3) = \boxed{.0134}$

BINOM option 1 $k=3$

or binompdf(5,.12,3) $= \boxed{.0134}$

Indep ✓ ✓

(b) only the first 3 restaurants fail the inspection?

Let F_1 be the event the 1st restaurant fails inspection

F_2 — — 2nd — — —.

etc.

$P(F_1 \cap F_2 \cap F_3 \cap F_4^c \cap F_5^c) = P(F_1)P(F_2)P(F_3)P(F_4^c)P(F_5^c)$

$=(.12)(.12)(.12)(1-.12)(1-.12)$

$=(.12)^3(.88)^2 = \boxed{.0013}$

(c) at least 4 of the restaurants pass the inspection?

← 4 or more

$n=5$

"success" – restaurant passes inspection

$p=.88$

Indep ✓

$P(4 \le X \le 5) = \boxed{.8875}$

BINOM option 2 lower $k=4$ upper $k=5$

$\boxed{1-\text{binomcdf}(5,.88,3)}$

(d) If 250 restaurants are inspected, how many can you expect to pass inspection? What would be the standard deviation of the number of restaurants that pass inspection?

$E(X) = np = (250)(.88) = \boxed{220}$

$\sigma_x = \sqrt{npq} = \sqrt{(250)(.88)(1-.88)}$

$\sigma_x = \boxed{5.1381}$

5. George is a hunter who prepares for hunts by shooting at a target. He hits the target with 83% of his shots. If he fires at the target 30 times,

(a) what is the probability that he hits the target at least 20 times but fewer than 27 times?

$n=30$

"success" = hit the target

$p=.83$

$P(20 \le X \le 26) = \boxed{.7670}$

$= \text{binomcdf}(30,.83,26) - \text{binomcdf}(30,.83,19)$

$= \boxed{.7670}$

(b) what is the probability that he will miss the target at least 10 times?

$n=30$

"success" = miss the target

$p = 1-.83 = .17$

$P(10 \le X \le 30) = \boxed{.0224}$

$\boxed{1-\text{binomcdf}(30,.17,9)}$

(c) How many times can you expect him to hit the target?

$E(X) = np = (30)(.83) = 24.9$

You can expect about 25 hits.

(d) What assumptions did you have to make in the above question to be able to use the binomial distribution to calculate the probabilities? Do you think these assumptions are justified?

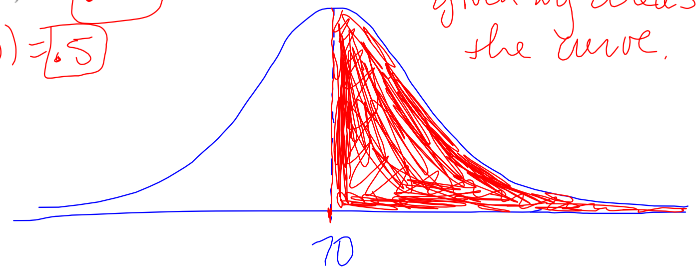
We had to assume that trials are independent and that the probability of him hitting the target stays exactly the same with each shot. In real life, it is possible that the hunter can improve his aim with each shot, so perhaps this assumption is not justified.

6. Let X be a normal random variable with $\mu = 70$ and $\sigma = 4$. By first sketching a normal curve and shading an appropriate area under the curve, find each of the following probabilities.

Probabilities are given by areas under the curve.

(a) $P(X > 70)$ $= \boxed{.5}$

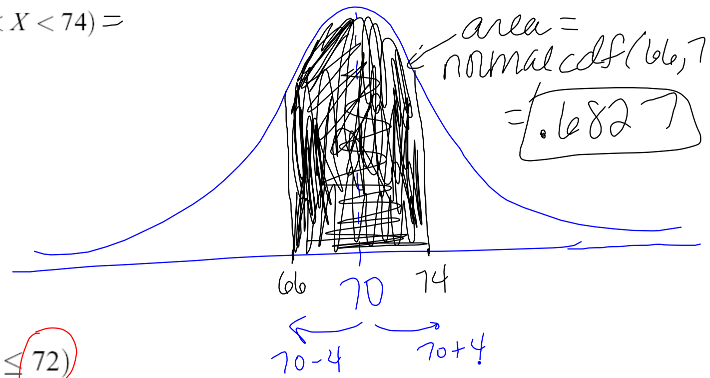
$P(X \geq 70) = \boxed{.5}$



(b) $P(66 < X < 74) =$

area = normalcdf(66, 74, 70, 4)

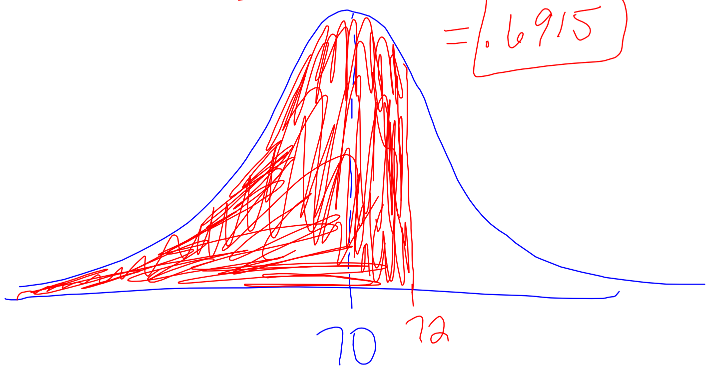
$= \boxed{.6827}$



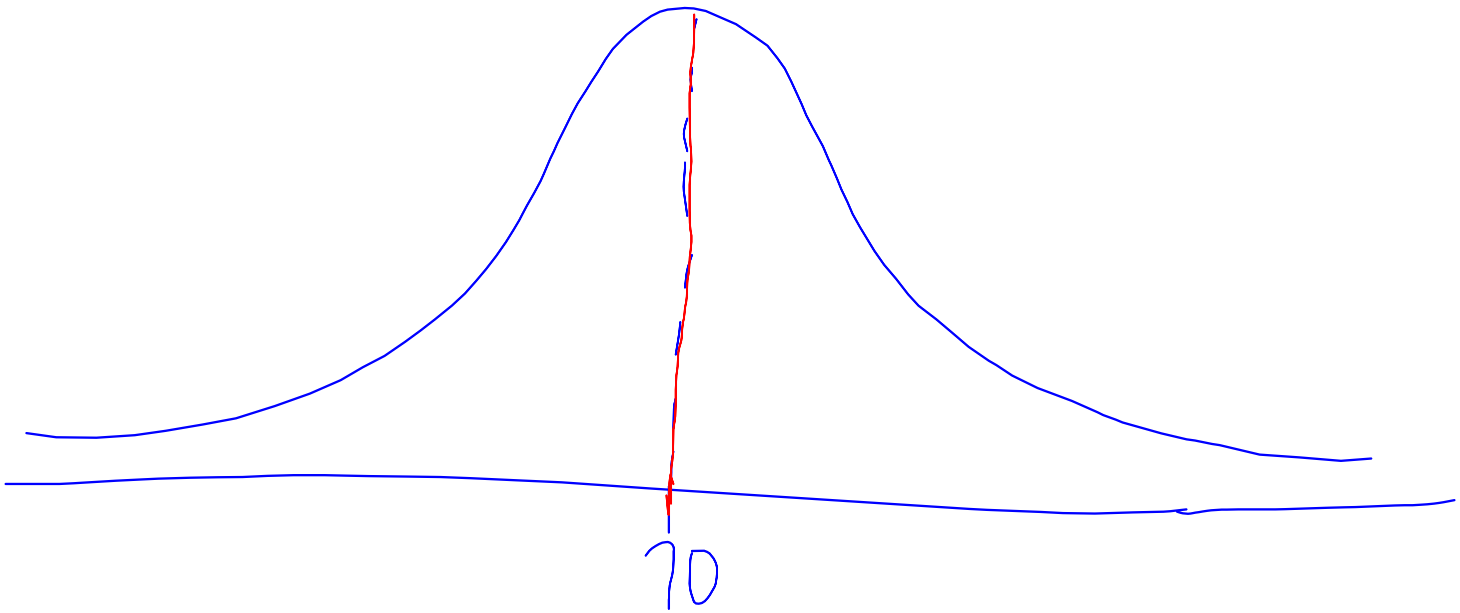
(c) $P(X \leq 72)$

area = $P(X \leq 72)$ = normalcdf(-1E99, 72, 70, 4)

$= \boxed{.6915}$



$$P(X = 70) = 0$$

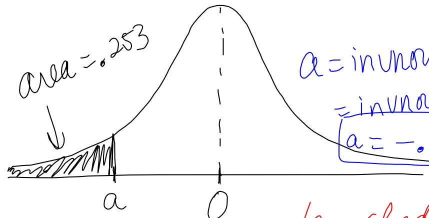


7. Let Z be the standard normal random variable. Find the value of a if

$\mu = 0$, $\sigma = 1$

(a) $P(Z < a) = 0.253$

we know the probability.



$a = \text{invnorm}(.253, 0, 1)$

$= \text{invnorm}(.253)$

$\boxed{a = -.66651}$

$a = \text{invnorm}(\text{area shaded to the left}, \mu, \sigma)$

(b) $P(Z < a) = 0.937$

shade left

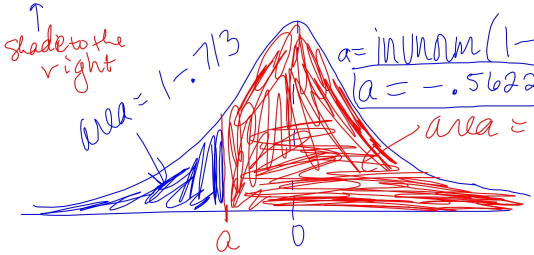
area = .937

$a = \text{invnorm}(.937, 0, 1)$

$\boxed{a = 1.5301}$

(c) $P(Z \geq a) = 0.713$

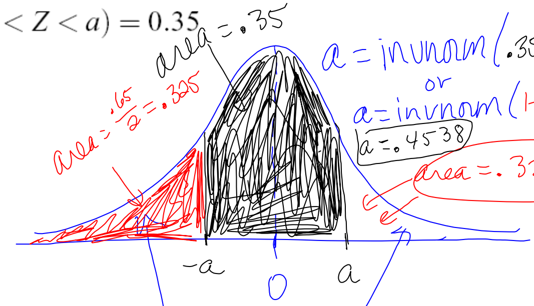
shade to the right



$a = \text{invnorm}(1 - .713, 0, 1)$

$\boxed{a = -.5622}$

(d) $P(-a < Z < a) = 0.35$



$a = \text{invnorm}(.35 + .325, 0, 1)$

or

$a = \text{invnorm}(1 - .325, 0, 1)$

$\boxed{a = .4538}$

areas total $1 - .35 = .65$

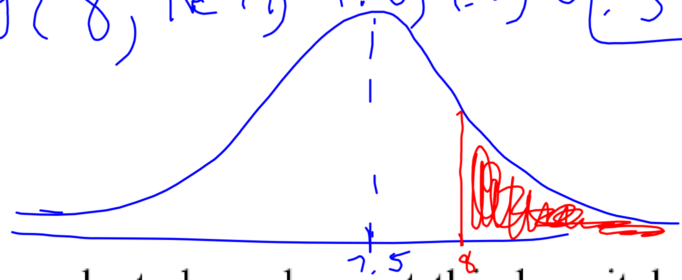
8. At a certain hospital, the weights of babies at birth are normally distributed with a mean of 7.5 pounds and a standard deviation of 1.1 pounds.

← normal curve

X = weight of baby

(a) What is the probability that a randomly selected newborn at this hospital weighs more than 8 pounds?

$P(X > 8) = \text{normalcdf}(8, 1E99, 7.5, 1.1) = \boxed{.3247}$



(b) What is the probability that a randomly selected newborn at this hospital weighs between 5 and 6 pounds?

$P(5 \le X \le 6) = \text{normalcdf}(5, 6, 7.5, 1.1) = \boxed{.0748}$

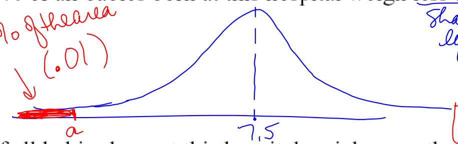
(c) What is the probability that a randomly selected newborn at this hospital weighs exactly 7.5 pounds?

$P(X = 7.5) = 0$

(d) Only 1% of all babies born at this hospital weigh less than $\underline{a}$ pounds.

1% of the area (.01)

shade left



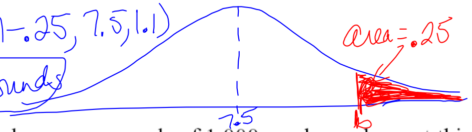
$a = \text{invnorm}(.01, 7.5, 1.1)$

$\boxed{a = 4.9410 \text{ pounds}}$

(e) 25% of all babies born at this hospital weigh more than $\underline{b}$ pounds.

$b = \text{invnorm}(1-.25, 7.5, 1.1)$

$\boxed{b = 8.2419 \text{ pounds}}$



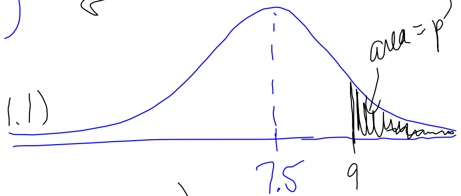
(f) If you randomly access records of 1,000 newborns born at this hospital, how many of those babies would you expect to weigh more than 9 pounds at birth?

$E(X) = np$

$n = 1000$

"success" = weighing more than 9 pounds

$p = P(X > 9)$



$p = \text{normalcdf}(9, 1E99, 7.5, 1.1)$

$p = .08634$

$E(X) = np = (1000)(.08634)$

$E(X) = 86.34$

About 86 babies will weigh more than 9 pounds.

9. A fair die is cast 2,500 times. What is the probability that an odd number lands up

since we have such a large # of trials, we will approximate the binomial distribution with the normal distribution.

(a) more than 1200 times?

$n = 2500$

"success" = odd lands up

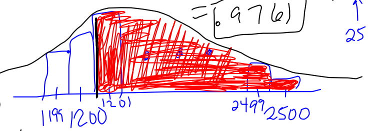
$p = \frac{1}{2}$

Indep? (yes)

$\mu = np = (2500)(\frac{1}{2}) = 1250$

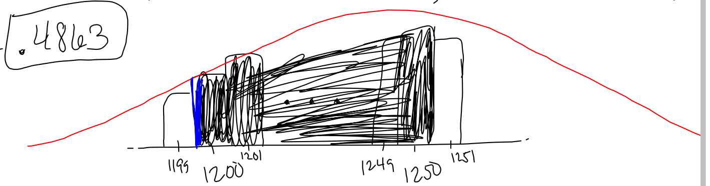
$\sigma = \sqrt{npq} = \sqrt{2500(\frac{1}{2})(\frac{1}{2})} = \sqrt{625} = 25$

$P(X > 1200) \approx \text{normalcdf}(1200.5, 2500.5, \mu, \sigma) = \boxed{.9761}$



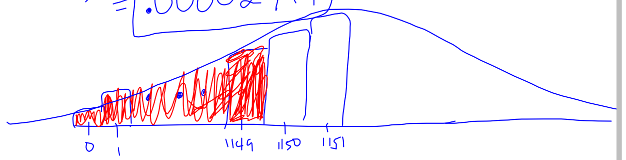
(b) between 1200 and 1250 times, inclusive?

$P(1200 \leq X \leq 1250) \approx \text{normalcdf}(1199.5, 1250.5, 1250, 25) = \boxed{.4863}$



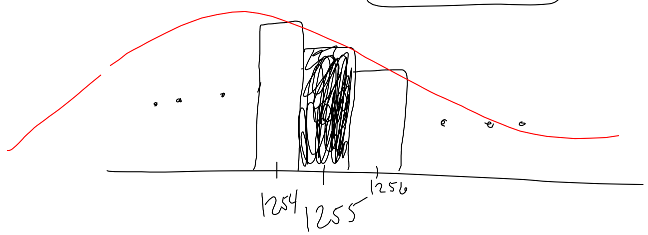
(c) fewer than 1150 times?

$P(X < 1150) \approx \text{normalcdf}(-.5, 1149.5, 1250, 25) = \boxed{.00002911}$



(d) exactly 1255 times?

$P(X = 1255) = \text{normalcdf}(1254.5, 1255.5, 1250, 25) = \boxed{.0156}$



10. Fun Trip Ships, Inc. has determined that 7% of the people who book passage on one of their cruises do not arrive for check-in at embarkation. The *Rey del Sol* cruise ship can accommodate 1,320 passengers. If Fun Trip Ships, Inc. books reservations for 1,400 passengers on this ship, what is the probability that the cruise is overbooked? Use the normal approximation to the binomial distribution.

X = # of passengers who check in.

$P(X > 1320) = P(X \geq 1321)$

$\approx \text{normalcdf}(1320.5, 1400.5, 1400 * .93, \sqrt{1400(.93)(.07)})$

$= \boxed{.0263}$

$n = 1400$

"success" – checking in

$p = 1 - .07 = .93$