# Math 166 - Week in Review #10

## Section 8.5 - The Normal Distribution

- Properties of the Normal Curve

  1. The normal curve is completely determined by $\mu$ and $\sigma$. ($\sigma$ determines the sharpness or flatness of the curve.)
  2. The curve has a peak at $x = \mu$.
  3. The curve is symmetric with respect to the vertical line $x = \mu$.
  4. The curve always lies above the $x$-axis but approaches the $x$-axis as $x$ extends indefinitely in either direction.
  5. The area under the curve and above the $x$-axis is 1.
  6. For any normal curve, 68.27% of the area under the curve lies within 1 standard deviation from the mean, 95.45% of the area lies within 2 standard deviations of the mean, and 99.73% of the area lies within 3 standard deviations of the mean.

- The *standard* normal random variable $Z$ has mean 0 and standard deviation 1.

## Section 8.6 - Applications of the Normal Distribution

- When approximating binomial probabilities by using the normal curve, first draw and shade a piece of a histogram corresponding to the probability you are being asked to find, and then use appropriate lower and upper bounds (adjust by 0.5) under the normal curve with $\mu = np$ and $\sigma = \sqrt{npq}$ to approximate the probability.

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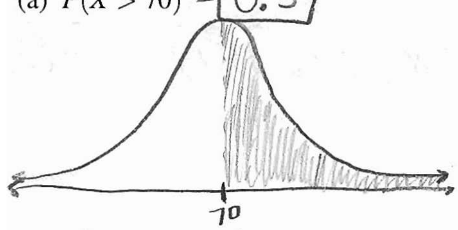
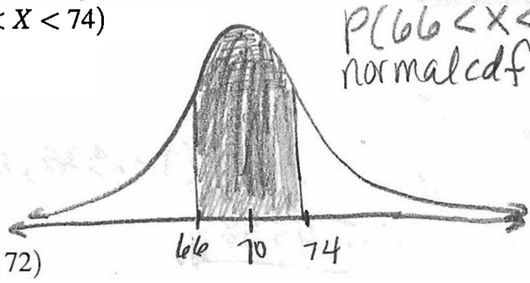
1. Let $X$ be a normal random variable with $\mu = 70$ and $\sigma = 4$. By first sketching a normal curve and shading an appropriate area under the curve, find each of the following probabilities.

   (a) $P(X > 70) = \boxed{0.5}$

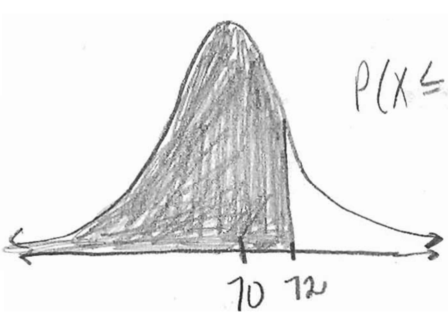



   (b) $P(66 < X < 74)$




$P(66 < X < 74) =$ normalcdf$(66, 74, 70, 4) = \boxed{.6827}$

   (c) $P(X \leq 72)$

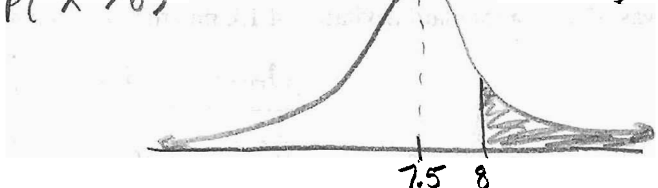



$P(X \leq 72) =$ normalcdf$(-1E99, 72, 70, 4) = \boxed{.6915}$

3. At a certain hospital, the weights of babies at birth are normally distributed with a mean of 7.5 pounds and a standard deviation of 1.1 pounds. $X =$ weight (in pounds) of an infant at birth)

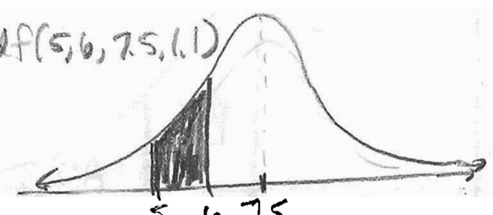
   (a) What is the probability that a randomly selected newborn at this hospital weighs more than 8 pounds?

   $P(X>8) = \text{normalcdf}(8, 1E99, 7.5, 1.1) = \boxed{0.3247}$



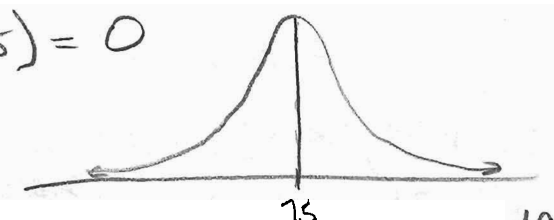

   (b) What is the probability that a randomly selected newborn at this hospital weighs between 5 and 6 pounds?

   $P(5 < X < 6) = \text{normalcdf}(5, 6, 7.5, 1.1)$
   $= \boxed{.0748}$



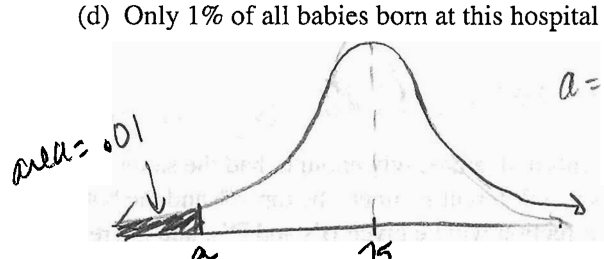

   (c) What is the probability that a randomly selected newborn at this hospital weighs exactly 7.5 pounds?

   $P(X = 7.5) = 0$



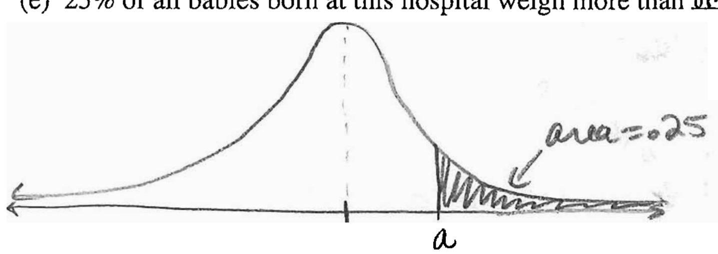

   (d) Only 1% of all babies born at this hospital weigh less than 4.9410 pounds.

   $a = \text{invnorm}(.01, 7.5, 1.1) = \boxed{4.9410}$



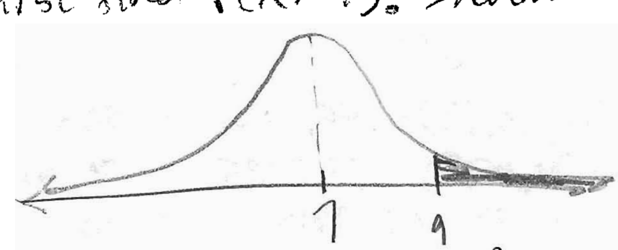

   (e) 25% of all babies born at this hospital weigh more than 8.2419 pounds.

   $a = \text{invnorm}(1-.25, 7.5, 1.1)$
   $\boxed{a = 8.2419}$




   (f) If you randomly access records of 1,000 newborns born at this hospital, how many of those babies would you expect to weigh more than 9 pounds at birth?

   First find $P(X>9)$: $= \text{normalcdf}(9, 1E99, 7.5, 1.1) = 0.0863$



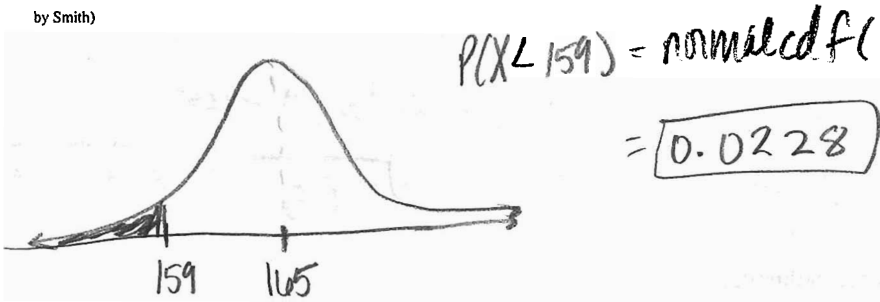

   $1000 \times .0863 = 86.3$

   physical interpretation:
   $\boxed{\text{86 babies}}$

6. The breaking strength (in pounds) of a certain new synthetic material is normally distributed with a mean of 165 and a variance of 9. The material is considered to be defective if the breaking strength is less than 159 pounds. What is the probability that a randomly selected piece of this material will be defective? (From *The Nature of Mathematics*, 10th ed., by Smith)

$P(X < 159) = \text{normalcdf}(-1E99, 159, 165, \sqrt{9})$

$= \boxed{0.0228}$



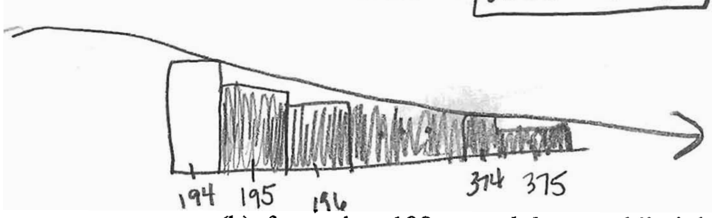

7. In the large city of Winchestertonfieldville, 40% of the drivers exceed the speed limit by 20 mph or more. Use the normal approximation to the binomial distribution for each of the following. Find the probability that among 375 drivers,

X = # of drivers who exceed limit by at least 20 mph.

(a) at least 195 exceed the speed limit by at least 20 mph.

$P(X \geq 195) \approx \text{normalcdf}(194.5, 375.5, 375(.4), \sqrt{375(.4)(1-}\,)$

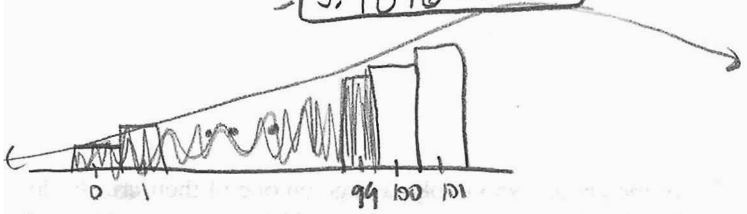
$= \boxed{.0000001363} = 1.363 \times 10^{-8}$




(b) fewer than 100 exceed the speed limit by 20 mph or more.

$P(X < 100) = \text{normalcdf}(-0.5, 99.5, 375(.4), \sqrt{375(.4)(.6)})$

$= \boxed{5.1096 \times 10^{-8}}$

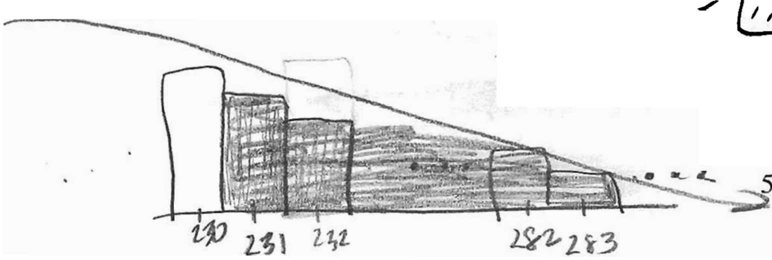



(c) between 231 and 283 drivers do not exceed the speed limit by 20 mph or more.

Y = # who don't exceed speed limit by 20 mph or more

$P(231 \leq X \leq 283) = \text{normalcdf}(230.5, 283.5, 225, \sqrt{90})$

$= \boxed{.2810}$




$\mu = np = (375)(\text{probability of not exceeding limit})$

$= (375)(.6)$

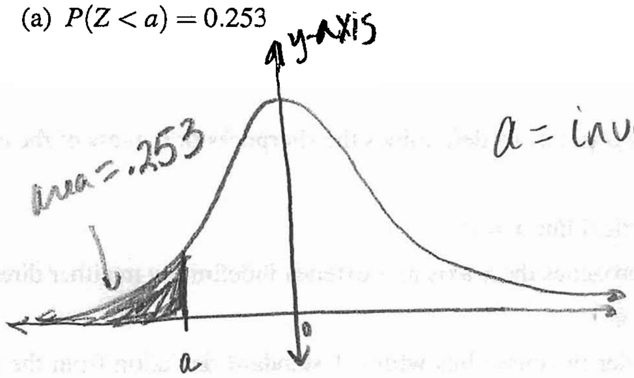
$= 225$

$\sigma = \sqrt{(375)(.6)(.4)} = \sqrt{225 \times .4} = \sqrt{90}$

$\mu = 0, \sigma = 1$

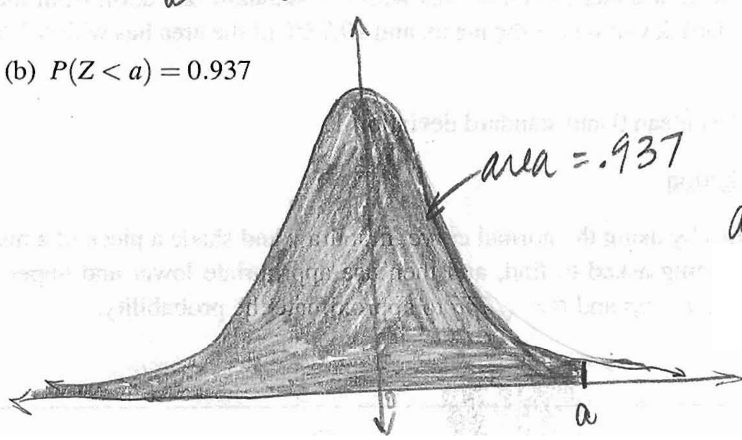
2. Let $Z$ be the standard normal random variable. Find the value of $a$ if

(a) $P(Z < a) = 0.253$



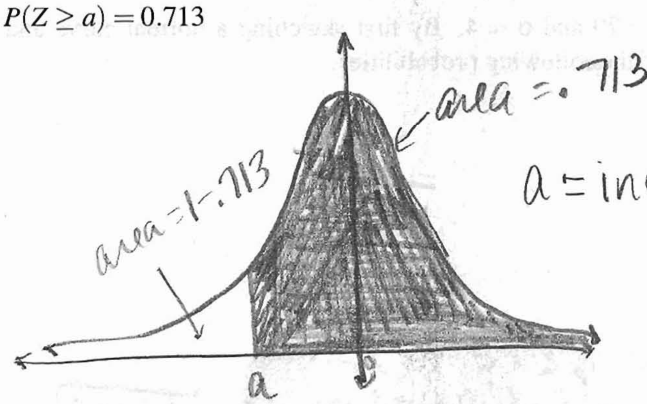

$a = \text{invnorm}(.253, 0, 1) = \boxed{-0.6651}$

(b) $P(Z < a) = 0.937$




$a = \text{invnorm}(.937) = \boxed{1.5301}$

(c) $P(Z \geq a) = 0.713$

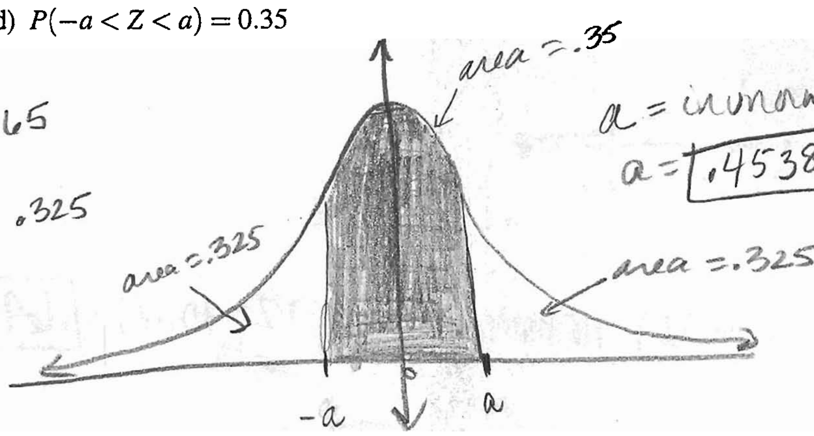



$a = \text{invnorm}(1 - .713) = \boxed{-0.5622}$

(d) $P(-a < Z < a) = 0.35$

$1 - .35 = .65$

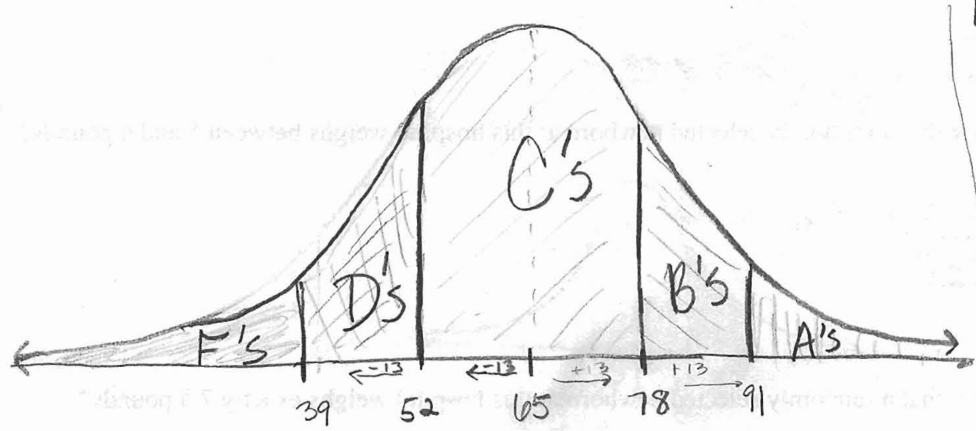
$.65/2 = .325$




$a = \text{invnorm}(1 - .325, 0, 1)$

$a = \boxed{.4538}$

4. An instructor of a physics class grades his exams according to the normal curve as follows: All students whose grades are within 1 standard deviation of the mean receive a C. Students whose grades are at least 1 standard deviation above the mean but less than 2 standard deviations above the mean receive a grade of B, and all students whose grades are at least 2 standard deviations above the mean receive an A. Students with grades in the range $\mu - 2\sigma \leq X < \mu - \sigma$ (where $X$ represents a student's exam score) receive a D, and all others below this receive an F. If 175 students took the exam, and the average was 65 with a standard deviation of 13, find the expected number of A's, B's, C's, D's, and F's.




Now find expected counts:

A: (.0228)(175) = 4

B: (.1359)(175) = 24

C: (.6827)(175) = 119

D: 24

F: 4 } by symmetry

First find probabilities of getting A, B, C, etc:

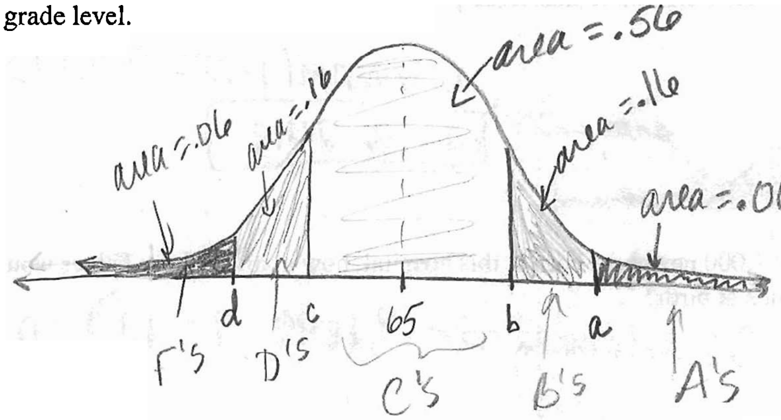
A → $P(X \geq 91) = \text{normalcdf}(91, 1E99, 65, 13) = 0.0228$

$P(78 \leq X < 91) = \text{normalcdf}(78, 91, 65, 13) = 0.1359$

C → $P(52 \leq X < 78) = .6827$ (1 std. dev)

D $P(39 \leq X < 52) = 0.1359$ (by symmetry) and F → $P(X < 39) = .0228$ (by symmetry)

5. A different physics instructor gave the same exam to her students and, amazingly enough, had the same average of 65 and standard deviation of 13. She decided to assign grades in a different manner: the top 6% and the bottom %6 will receive A's and F's, respectively. The next 16% in either direction will be given B's and D's, and the remaining students will receive C's. Assuming that the grades on the exam are normally distributed, find the cutoffs for each grade level.




Cutoff for A's = a = invnorm(1 - .06, 65, 13) = 85.2121 ≈ 85 & higher = A

Cutoff for B's = b = invnorm(1 - .16 - .06, 65, 13) = 75.0385 ≈ 75 to 84 = B

Cutoff for C's = c = invnorm(.06 + .16, 65, 13) = 54.9615 ≈ 55 to 74 = C

Cutoff for D's = d = invnorm(.06, 65, 13) = 44.7879 ≈ 45 to 54 = D

44 and lower = F

 Very lg # of trial, use Normal Approx. 

8. A fair die is cast 2,500 times. What is the probability that an odd number lands up

(a) more than 1200 times?

X = # of odds

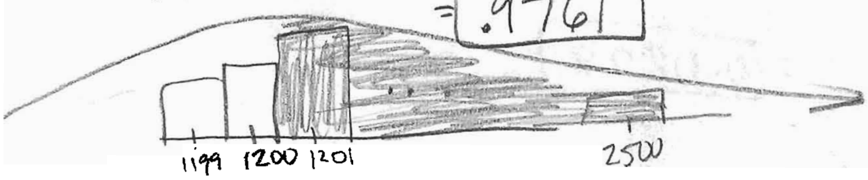
$P(X > 1200) = $ normalcdf(1200.5, 2500.5, 1250, 25)

$= \boxed{.9761}$

$\mu = 2500(.5) = 1250$

$\sigma = \sqrt{npq} = \sqrt{1250(.5)} = \sqrt{625}$

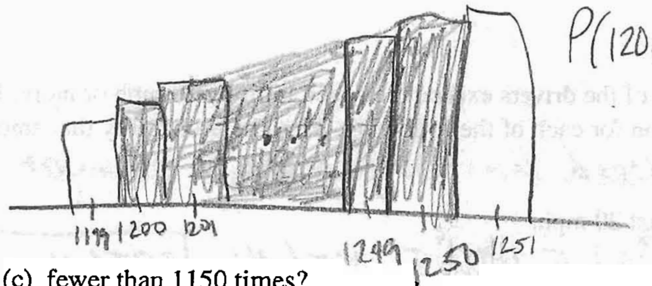
$= 25$




(b) between 1200 and 1250 times, inclusive?

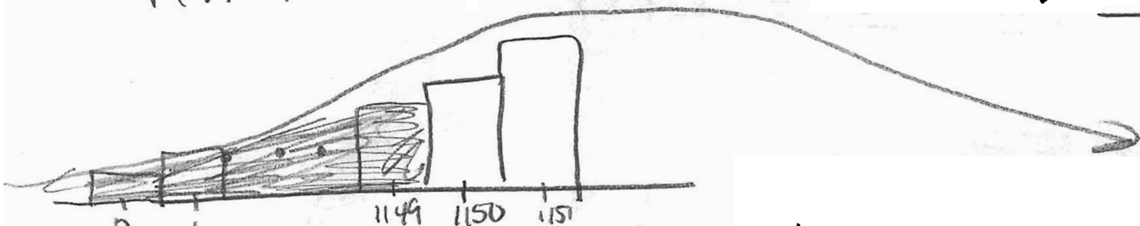
$P(1200 \le X \le 1250) \approx$ normalcdf(1199.5, 1250.5, 1250, 25)

$= \boxed{.4863}$




(c) fewer than 1150 times?

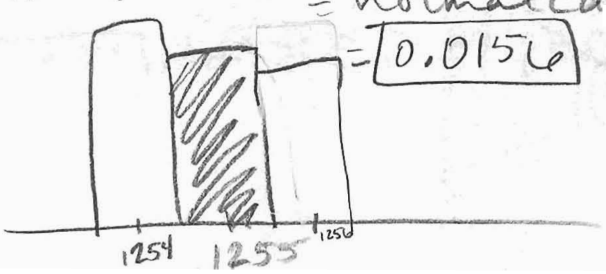
$P(X < 1150) \approx$ normalcdf(-.5, 1149.5, 1250, 25)




(d) exactly 1255 times?

= normalcdf(1254.5, 1255.5, 1250, 25)

$= \boxed{0.0156}$




9. Fun Trip Ships, Inc. has determined that 7% of the people who book passage on one of their cruises do not arrive for check-in at embarkation. The *Rey del Sol* cruise ship can accommodate 1,320 passengers. If Fun Trip Ships, Inc. books reservations for 1,400 passengers on this ship, what is the probability that the cruise is overbooked? Use the normal approximation to the binomial distribution.

Over booked means more than 1320 arrive at checkin.

success

n = 1400

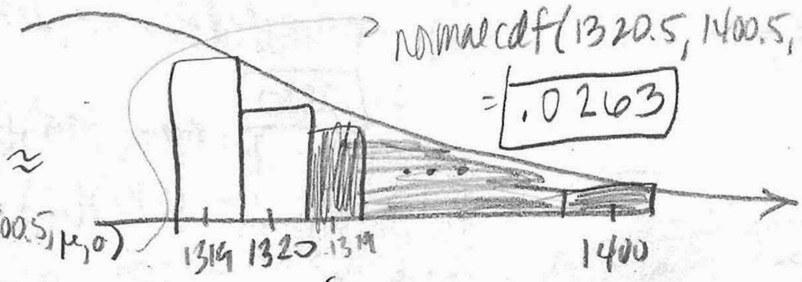
p = 1 - .07 = .93

$P(X > 1320) \approx$

normalcdf(1320.5, 1400.5, $\mu$, $\sigma$)

→ normalcdf(1320.5, 1400.5, 1302, $\sqrt{91.14}$)

$= \boxed{.0263}$




$\mu = 1400(.93) = 1302$

$\sigma = \sqrt{1400(.93)(1-.93)} = \sqrt{91.14}$