Math 166 - Week in Review #10

Chapter F - Finance

- **Simple Interest** - interest that is computed on the original principal only
- Simple Interest Formulas

  Interest $= I = Prt$

  Accumulated Amount $= A = P + I = P + Prt = P(1 + rt)$

  NOTATION: I = interest earned, P = principal, r = interest rate (as a decimal), t = term of the investment in ***YEARS***, A = accumulated amount
- **The TVM-Solver CANNOT be used for simple interest calculations.**
- **Compound Interest** - earned interest that is periodically added to the principal and thereafter itself earns interest at the same rate.
- The TVM-Solver can be used in problems involving compound interest as follows:

  N = total number of payments made, usually $m \times t$.

  I% = interest rate in *percent form*. Don't convert to decimal form!!

  PV = present value (prinicpal, or the amount you start with). Entered as negative if invested, positive if borrowed.

  PMT = payment (amount paid each period). Entered as negative if paying off a loan, positive if receiving money, 0 if computing compound interest.

  FV = future value (accumulated amount). This will be 0 if paying off a loan.

  P/Y = number of payments per year (usually the same as m).

  C/Y = number of conversions per year (m).
- At the bottom of the screen, you will see PMT:END BEGIN. If END is highlighted, then the TVM Solver calculates everything with payments being made at the end of the period. For virtually all of the problems we will work in class, END should be highlighted.
- You can solve for any quantity on the TVM-Solver by moving the cursor to that quantity and then pressing ALPHA followed by ENTER.
- **Effective Rate of Interest** - The effective rate of interest is a way of comparing interest rates. More precisely, the *effective rate* is the simple interest rate that would produce the same accumulated amount in 1 year as the nominal rate compounded m times per year.
- The effective rate of interest is typically denoted by r_{eff} and is also known as the effective annual yield.
- To calculate the effective rate of interest, use the Eff( ) function on the calculator. This function can be found under Finance–just arrow down until you see C: Eff(.
- The Eff( ) function has two parameters, the nominal (or annual) interest rate entered as a percent, and the number of conversion, m, per year: Eff(nominal rate as a percent, m)
- **Annuity** - a sequence of payments made at regular time intervals.
- In this course, we will study annuities with the following properties:
  1. The terms are given by fixed time intervals.
  2. The periodic payments are equal in size.
  3. The payments are made at the end of the payment periods.
  4. The payment periods coincide with the interest conversion periods.

1. Jake deposited \$350 into an account paying 3.25% simple interest. How much money is in the account at the end of 4 years? How much interest was earned?

Interest earned $= I = Prt = 350(.0325)(4) = \boxed{\$45.50}$

Accumulated Amount after 4 years $= P + I = 350 + 45.50 = \boxed{\$395.50}$

2. When Erica graduated from high school, she received \$500 from her parents as a gift. She then loaned this money to her brother who repaid her 3 months later with a sum of \$510.25. What was the simple interest rate that Erica charged her brother?

$I = Prt$

$10.25 = 500 * r * \frac{3}{12}$

$r = 10.25/(500*3/12) = 0.082$

$\boxed{8.2\%}$

3. Annette wants to take a trip to Europe when she graduates. She will need \$4,500 for this trip.

(a) How much money should Annette deposit now into an account paying 8%/year compounded quarterly if she expects to graduate in 4 years?

TVM Solver

$N = 4*4$ $PMT = 0$

$I\% = 8$ $FV = 4500$

$PV = ?$ $P/Y = C/Y = 4$

$\boxed{\text{Deposit } \$3278.01}$

(b) How much interest will she earn in the 4 years?

Interest earned $= 4500 - 3278.01$

$= \boxed{\$1221.99}$

(c) How much interest will she earn in the third quarter of the second year?

1st year = 4 quarters

2nd year = 1st, 2nd, 3rd, 4th
5th quarter, 6th quarter, 7th quarter

Interest earned in 3rd quarter of 2nd year equals the difference in account balances b/w the 6th and 7th quarters (since no payment being added)

Account Balance after 6 quarters

$N = 6$ $PMT = 0$

$I\% = 8$ $FV = ? \to \$3691.57$

$PV = -3278.01$ $P/Y = C/Y = 4$

Account Balance after 7 quarters

$N = 7$ $PMT = 0$

$I\% = 8$ $FV = ? \to \$3765.40$

$PV = -3278.01$ $P/Y = C/Y = 4$

Interest Earned

$3765.40 - 3691.57$

$= \boxed{\$73.83}$

4. Lynn, Annette's twin sister, wants to take that same trip to Europe, but she does not have enough money to open the same type of account as Annette. Instead, she plans to make monthly payments to an account paying 8.25%/year compounded monthly.

   (a) How much should each payment be so that she has $4,500 at the end of 4 years?

   N = 12 * 4
   I% = 8.25
   PV = 0
   PMT = ? $\boxed{\$79.45}$
   FV = 4500
   P/Y = C/Y = 12

   (b) How much interest will Lynn earn?

   Lynn's totae deposit = 79.45 * 12 * 4 = $3813.60

   Interest earned = 4500 − 3813.60 = $\boxed{\$686.40}$

5. Kira opened an account paying 5.25%/year compounded monthly with $100 and plans to add $50 at the end of each month until she has at least $45,000.

   (a) How long will it take her to first reach her goal?

   N = ?
   I% = 5.25
   PV = −100
   PMT = −50
   FV = 45000
   P/Y = C/Y = 12

   N = 363.7185678
   so 363 payments is not enough – she needs 364 payments (months)
   364/12 = $30\frac{1}{3}$ years.

   (b) How much will she actually have in the account when she first reaches her goal?

   N = 364
   I% = 5.25
   PV = −100
   PMT = −50
   FV = ? → $\boxed{\$45{,}049.64}$
   P/Y = C/Y = 12

   (c) How much interest will Kira earn in the third month of her sixth year of making this investment?

   5 yrs = 5 * 12 payments = 60 payments

   6th yr: 1st 2nd 3rd 4th 5th 6th . . . month
   61 62 63 64 65 66 .. months

① Find FV for 62 months and FV for 63 months:

After 62 months
N = 62 PMT = −50
I% = 5.25 FV = ? → $3683.39
PV = −100 P/Y = C/Y = 12

After 63 Months
N = 63 PMT = −50
I% = 5.25 FV = ? → $3749.51
PV = −100 P/Y = C/Y = 12

② Change in account value for 63rd month:
3749.51 − 3683.39
= $66.12
But $50 of this was a payment!

③ Subtract Pmt
$66.12 − $50
= $\boxed{\$16.12}$
Interest earned 3rd mo. of 6th yr.

6. Benjamin is 25 years old and plans to retire in 40 years. When he retires, he would like to receive monthly payments of \$3,000 from a retirement account for 15 years.

   (a) How much money should Benjamin deposit at the end of each month from now until he retires to achieve this goal if he secures an account that will pay 6.25%/year compounded monthly for the life of the account?

Step1: Find starting amount needed for retirement account.

N = 12*15
I% = 6.25
PV = ? → \$349885.70
PMT = 3000
FV = 0
P/Y = C/Y = 12

So start retirement account with \$349885.70 at retirement

Step2: Find monthly pmt

N = 12*40
I% = 6.25
PV = 0
PMT = ? $\boxed{\$164.12}$
FV = 349885.70
P/Y = C/Y = 12

   (b) How much will Benjamin deposit into this account?

164.12 * 12 * 40 = $\boxed{\$78,777.60}$

   (c) How much interest will be earned over the entire life of the account?

Two steps: ① Find interest earned while saving for retirement
② Find interest earned while withdrawing from retirement account.

① While Saving

349,885.70 − 78777.60
= \$271,108.10

② While withdrawing

Total withdrawal − starting amount
= 3000*12*15 − 349885.70
= \$190114.30

Total Interest Earned
\$271108.10 + 190114.30
= $\boxed{\$461,222.40}$

7. Julian opened an account with \$8,000 and after 7 years, it had grown to \$10,000.

   (a) What was the annual interest rate if interest was compounded weekly?

N = 52*7
I% = ?
PV = −8000
PMT = 0
FV = 10000
P/Y = C/Y = 52

I% = $\boxed{3.1887\%}$

   (b) If the annual interest rate found in part (a) was instead a simple interest rate, how long would it take for Julian's \$8,000 to grow to \$10,000?

3.1887% simple interest

I = Prt

2000 = 8000 * 0.031887 * t

$\boxed{7.8402 \text{ yrs} = t}$

8. Miles and Keiko are shopping for a new home. They can afford a down payment of $25,000 and monthly payments of at most $850. Bank A has offered to finance a loan at 8.75%/year compounded monthly for 30 years, whereas Bank B has offered 8.25%/year compounded monthly for 25 years.

   (a) What is the most expensive house they can afford to buy? Which bank would they have to use for this house?

Bank A

N=12*30 PMT=-850

I%=8.75 FV=0

PV=? P/Y=C/Y=12

→ Loan of $108046.21

With Bank A, they can afford a house valued at $25,000+ 108046.21 = $133046.21

Bank B

N=12*25 PMT= -850

I%=8.25 FV=0

PV= ? P/Y=C/Y=12

→ Loan of $107806.44

with Bank B, they can afford $25,000 + 107806.44 = $132806.44

The most expensive house they can afford is $133046.21 using Bank A's financing plan.

   (b) Miles and Keiko ultimately make a down payment of $25,000 on a $110,000 home and finance the balance through Bank B. What monthly payments should they make to pay off the house in 25 years? How much interest did they pay?

N = 12*25 PMT= ? → $\boxed{\$670.18 \text{ monthly pmt}}$

I% = 8.25 FV=0

PV = 85000 P/Y=C/Y=12

Total amt paid on loan = 670.18*12*25 = $201,054

Interest paid = 201,054 - 85,000 = $\boxed{\$116,054}$

(or Total amt pd on house = $25000 + 670.18*12*25 = $226,054

Interest pd = 226054 - 110000 = $\boxed{\$116,054}$)

   (c) Refering to part (b), create an amortization schedule for the first 4 months of the loan.

period	interest owed	payment	amount toward principal	outstanding principal
0	——	——	——	85000
1	584.38	670.18	85.80	84914.20
2	583.79	670.18	86.39	84827.81
3	583.19	670.18	86.99	84740.82
4	582.59	670.18	87.59	84653.23

85000 * .0825/12 = 584.38

9. If Bank A has a savings account paying 8%/year compounded semiannually and Bank B offers 7.9%/year compounded monthly, which is the better offer?

Bank A: Eff(8, 2) = 8.16%

Bank B: Eff(7.9, 12) = 8.1924%

Bank B is the better offer since it has the higher effective rate.

10. Juanita decided to purchase a flat-screen HDTV. She makes a down payment of $250 and secures financing for the balance of the purchase price at a rate of 12%/year compounded monthly. Under the terms of the finance agreement, she is required to make monthly payments of $125 for 30 months.

(a) What was the cash price of the TV?

Find loan amt:

N = 30 PMT = −125

I% = 12 FV = 0

PV = ? P/Y = C/Y = 12

Loan Amt = $ 3225.96

Cash price = Down pmt + Loan amt

= 250 + 3225.96

= $\boxed{\$3475.96}$

(b) How much interest did Juanita pay?

Total amt paid on loan = 125 * 30 = $3750

Interest paid = 3750 − 3225.96 = $\boxed{\$524.04}$

(or total amt pd = 250 + 125 * 30 = 4000

Interest pd = 4000 − 3475.96 = $\boxed{\$524.04}$)

11. Deanna owes $1,000 on a credit card that has an interest rate of 22.5%/year compounded monthly. If she pays the minimum payment of $20 each month,

(a) how much of her first payment goes toward interest?

Interest owed = 1000 * 0.225/12 = $\boxed{\$18.75}$

(b) how long will it take her to pay off the card? (Assume no additional charges are made.)

N = ? PMT = −20

I% = 22.5 FV = 0

PV = 1000 P/Y = C/Y = 12

→ N = 149.2534 so 149 payments will not be enough — 150 pmts (months) needed = $\boxed{12.5\text{ years}}$

105000 = 80% of purchase price
10500 = .8(purchase price)
so purchase price = 131250

12. The Gardners purchased a vacation home 15 years ago. At the time of the purchase, they were able to make a down payment of 20% of the purchase price and then secured a loan of $105,000 to finance the remaining amount. The loan was to be amortized with monthly payments over 30 years at an interest rate of 6.75%/year compounded monthly.

(a) What is the current outstanding principal on the loan?

Step 1: Find monthly payment.

N = 12*30
I% = 6.75
PV = 105,000
PMT = ? → $681.03
FV = 0
P/Y = C/Y = 12

Step 2: Find outstanding principal

N = 12*15
I% = 6.75
PV = 105000
PMT = −681.03
FV = ?
P/Y = C/Y = 12

$\boxed{\text{still owe } \$76,959.57}$

(or bal(12*15) = ⊳)

(b) How much equity do the Gardners have in their vacation home?

Equity = Value of home − what you still owe
= 131,250 − 76959.57
= $\boxed{\$54,290.43}$

(c) Over the 30-year period, how much interest will the Gardners pay?

Amt pd on loan = $681.03 * 12 * 30 = $245170.80

Interest paid = 245170.80 − 105000
= $\boxed{\$140,170.80}$

13. Find the effective rate of 6% per year compounded semiannually.

Eff(6, 2) = $\boxed{6.09\%}$