Math 166 - Final Exam Review

The following review covers Sections L.1, L.2, 1.1-1.7, 2.1-2.4, 3.1-3.4, 4.3, 4.4, 5.1, 5.2, M.1-M.3, F.1-F.4, G.1, and G.2 and contains about 1 problem (or 1 part of a problem) per section. Since many sections contain multiple topics that can be addressed in separate problems, this review should not be used as your sole source of practice problems as you study for the final exam. You should also review all examples demonstrated in lecture, problems on the exams and quizzes you have taken this semester, the suggested homework problems from your textbook, the problems from the online homeworks, and problems from previous week-in-reviews.

1. An insurance company classifies drivers as low-risk if they are accident-free for 1 year. Past records indicate that 98% of the drivers in the low-risk category one year will remain in that category the next year, and 78% of the drivers who are not in the low-risk category one year will move to the low-risk category the next year. (pg. 598 of *Finite Mathematics for Business, Economics, Life Sciences, and Social Sciences* by Barnett, Ziegler, and Byleen)

   (a) If 90% of the drivers in the community are in the low-risk category this year, what is the probability that a driver chosen at random from the community will be in the low-risk category two years from now? X_2

   L - low risk $\quad L^c$ - Not low risk

   $$T = \begin{array}{cc} & \text{Current} \\ & \begin{array}{cc} L & L^c \end{array} \\ \text{Next} \begin{array}{c} L \\ L^c \end{array} & \begin{bmatrix} .98 & .78 \\ .02 & .22 \end{bmatrix} \end{array}, \quad X_0 = \begin{bmatrix} .9 \\ .1 \end{bmatrix} \qquad X_2 = T^2 X_0 = \begin{bmatrix} .972 \\ .028 \end{bmatrix}$$

   $$\boxed{0.972}$$

   (b) If this trend continues, what percent of the drivers in the community will be in the low-risk category in the long run?

   $TX = X$ where $X = \begin{bmatrix} x \\ y \end{bmatrix}$ is the steady-state distribution vector.

   $$\begin{bmatrix} .98 & .78 \\ .02 & .22 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

   $.98x + .78y = x$
   $.02x + .22y = y$

   $-0.02x + 0.78y = 0$
   $0.02x - 0.78y = 0$
   $x + y = 1$

   $$\left[\begin{array}{cc|c} -0.02 & 0.78 & 0 \\ 0.02 & -0.78 & 0 \\ 1 & 1 & 1 \end{array}\right] \xrightarrow{rref} \left[\begin{array}{cc|c} 1 & 0 & 0.975 \\ 0 & 1 & 0.025 \\ 0 & 0 & 0 \end{array}\right] \leftarrow \boxed{97.5\%}$$

2. Perform the next two row operations in the Gauss elimination method:

   $$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 3 \\ 0 & 5 & 6 & 0 \\ 0 & -4 & 3 & -1 \end{array}\right] \xrightarrow{R_2 + R_3 \to R_2} \left[\begin{array}{ccc|c} 1 & 0 & 2 & 3 \\ 0 & 1 & 9 & -1 \\ 0 & -4 & 3 & -1 \end{array}\right] \xrightarrow{R_3 + 4R_2 \to R_3} \left[\begin{array}{ccc|c} 1 & 0 & 2 & 3 \\ 0 & 1 & 9 & -1 \\ 0 & 0 & 39 & -5 \end{array}\right]$$

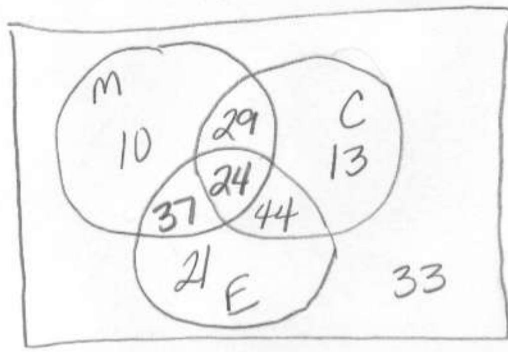
   make a 1

   make a 0

3. A survey of some college students was conducted to see which of the following three countries they had visited: Mexico, Canada, and England. It was found that

M - the set of those surveyed who had visited Mexico.

24 students had visited all three countries.
110 students had visited exactly 2 of these 3 countries.
44 students had visited exactly 1 of these 3 countries.
37 students had visited only Mexico and England.
53 students had visited Mexico and Canada.
126 students had visited England.
168 students had visited Canada or England.
46 students had visited neither Mexico nor England.



(a) How many students were surveyed?

$$10+29+13+37+24+44+21+33=\boxed{211}$$

(b) How many students surveyed had visited Canada and England?

$$24+44=\boxed{68}$$

(c) How many students surveyed had visited Mexico or Canada but not both?

$$10+37+13+44=\boxed{104}$$

(d) What is the probability that a randomly selected student from this survey had been to none of these three countries?

$$\boxed{\frac{33}{211}}$$

(e) What is the probability that a randomly selected student from this survey had not visited England but had been to at least one of the other two countries?

$$\frac{10+29+13}{211}=\boxed{\frac{52}{211}}$$

4. Classify each of the following types of random variables as either finite discrete, infinite discrete, or continuous, and state the possible values of the random variable.

(a) $X=$ the number of black cats being boarded at a kennel that can house at most 45 animals.

$X=0,1,2,\ldots,45$ finite discrete

(b) $Y=$ the number of gallons of water in a bucket that has a maximum capacity of 5 gallons.

$0\le Y\le 5$ continuous

(c) $Z=$ the number of times my phone rings before I answer it.

$Z=0,1,2,\ldots$ infinite discrete

5. Amelia wants to save enough money so that she will have \$3,000 to spend on a trip to Europe that she is planning to take in 5 years. If she opens an account paying 6% interest compounded monthly with \$400 and makes monthly deposits for 5 years, what is the size of the monthly payment that will reach her goal?

$N = 12 * 5$ $\quad PMT = ? \rightarrow \boxed{\$35.27}$

$I\% = 6$ $\quad FV = 3000$

$PV = -400$ $\quad P/Y = C/Y = 12$

6. In an experiment, several people are randomly surveyed to find out if they consider themselves to be a Republican, Democrat, or Independent. Each person's response, along with his or her sex, is recorded.

R - Republican
D - Democrat
I - Independent
M - Male
F - Female

(a) Write an appropriate sample space for this experiment.

$S = \{(R,M),(R,F),(D,M),(D,F),(I,M),(I,F)\}$

(b) Write the event that the person surveyed is female.

$E = \{(R,F),(D,F),(I,F)\}$

(c) Write the event that the person surveyed considers himself or herself to be an Independent.

$G = \{(I,M),(I,F)\}$

(d) Are the events found in (b) and (c) mutually exclusive?

Not mutually exclusive since E and G can happen at the same time: $E \cap G = \{(I,F)\}$.

7. Consider the propositions

p: Some of the presents are wrapped.
q: All of the guests have arrived.
r: The food is not ready.

(a) Write symbolically the statement, "All of the guests have arrived, but the food is not ready and none of the presents are wrapped."

$q \wedge (r \wedge \sim p)$

(b) Write symbolically the statement, "Either the food is ready or all of the guests have arrived, but not both."

$\sim r \veebar q$

(c) Write the statement $(\sim q \vee r) \wedge p$ in English.

Not all of the guests have arrived or the food is not ready, but some of the presents are wrapped.

(d) Write the statement $\sim (r \vee p)$ in English.

$\sim(r \vee p) = \sim r \wedge \sim p$

The food is ready (but) and none of the presents are wrapped.

8. The Capital Two credit card company has found that 23% of its cardholders are late in making their monthly payments. Find the probability that among 50 randomly selected cardholders, at least 10 but fewer than 30 are late in making their payment in a particular month.

X = # of successes

1. $n=50$
2. "success" – late in making payment
3. $p=.23$
4. Indep ✓

$P(10 \le X < 30) = \text{binomcdf}(50, .23, 29) - \text{binomcdf}(50, .23, 9)$

$= \boxed{0.7436}$

9. Fred, Bob, and Sue are all aliens with extremely long life expectancies. Bob and Sue's combined age in years is only half that of Fred's, and Bob has been alive for three times as many years as Sue. If the three friends' combined ages total 504 years, how old is each alien?

Let x = the age (in years) of Fred.
Let y = the age (in years) of Bob.
Let z = the age (in years) of Sue.

$$x+y+z=504 \qquad\qquad x+y+z=504$$
$$y+z=\tfrac{1}{2}x \quad \longrightarrow \quad -\tfrac{1}{2}x+y+z=0$$
$$y=3z \qquad\qquad y-3z=0$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 504 \\ -\frac{1}{2} & 1 & 1 & 0 \\ 0 & 1 & -3 & 0 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 336 \\ 0 & 1 & 0 & 126 \\ 0 & 0 & 1 & 42 \end{array}\right]$$

Fred is 336 years old, Bob is 126 years old, and Sue is 42 years old.

10. Six people get into an elevator that services 12 floors in a certain building. What is the probability that these 6 people exit the elevator with at least 2 people exiting on the same floor?

E – event that at least two exit on the same floor

$P(E) = 1 - P(E^c)$ where E^c – event that they all exit on different floors.

$$= 1 - \frac{n(E^c)}{n(S)} = 1 - \frac{12\cdot 11\cdot 10\cdot 9\cdot 8\cdot 7}{12\cdot 12\cdot 12\cdot 12\cdot 12\cdot 12} = 1 - \frac{P(12,6)}{12^6} = \boxed{\frac{1343}{1728}}$$

11. Construct the truth table for $\sim(p \wedge \sim q) \vee p$.

p	q	$\sim q$	$p\wedge\sim q$	$\sim(p\wedge\sim q)$	$\sim(p\wedge\sim q)\vee p$
T	T	F	F	T	T
T	F	T	T	F	T
F	T	F	F	T	T
F	F	T	F	T	T

Note: This is a tautology.

12. Bob just bought a house for \$200,000. He made a 15% down payment and financed the balance with a loan that has an interest rate of 5.45%/year compounded monthly.

    (a) If the loan is amortized for 25 years, what is Bob's monthly payment?

    $N = 12 * 25$ $\quad PMT = ? \rightarrow \boxed{\$1038.88}$

    $I\% = 5.45$ $\quad FV = 0$

    $PV = 170000$ $\quad P/Y = C/Y = 12$

    (b) After 10 years of payments, how much equity has Bob earned on his house?

    Equity = Value of house − what you still owe:

    $= 200000 - 127558.37$

    $= \boxed{\$72441.63}$

    $N = 12 * 10$ $\quad PMT = -1038.88$

    $I\% = 5.45$ $\quad FV = ?$

    $PV = 170000$ $\quad P/Y = C/Y = 12$

    $\$127558.37$

    (c) How much interest will Bob pay on the loan?

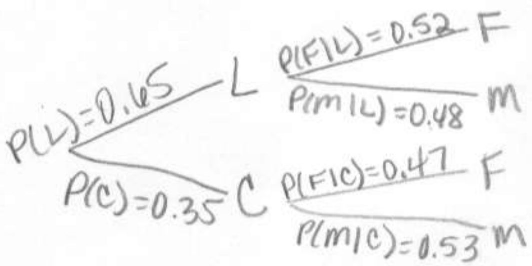
    $1038.88 * 12 * 25 - 170000$

    $= \boxed{\$141,664}$

13. At a certain university, 65% of the students are liberal arts majors, and 35% are science majors. Fifty-two percent of the liberal arts majors and 47% of the science majors are female.

    (a) What is the probability that a randomly selected student at this university is a science major if it is known that she is female?

    L - Liberal Arts major $\quad$ C - science major $\quad$ F - female $\quad$ M - male



$$P(C|F) = \frac{P(C \cap F)}{P(F)}$$

$$= \frac{(0.35)(0.47)}{(0.65)(0.52) + (0.35)(0.47)}$$

$$\approx \boxed{0.3274} \quad \text{or} \quad \frac{329}{1005}$$

    (b) What is the probability that a randomly selected student is male and a liberal arts major?

    $P(M \cap L) = (.65)(.48) = \boxed{0.312}$

    (c) Are the events of being a science major and being female independent? Why or why not?

    Test for Indep

    $P(C \cap F) \stackrel{?}{=} P(C)P(F)$

    $(.35)(.47) \stackrel{?}{=} (0.35)(0.5025)$

    $0.1645 \neq 0.1759$

    $P(F) = (0.65)(0.52) + (0.35)(0.47)$

    $= 0.5025$

    $\boxed{\text{Not independent}}$

14. If A is a 4×4 matrix, B is a 2×4 matrix, C is a 4×5 matrix, and D is a 5×4 matrix, which of the following are possible? If the operation IS possible, give the size of the resulting matrix. If the operation is NOT possible, explain why.

(a) BA

size of B: 2×4 $\checkmark$ size of A: 4×4 → 2×4

$\boxed{BA \text{ is } 2\times4}$

(b) $A^T C + D$

size of A^T: 4×4 $\checkmark$ size of C: 4×5 → 4×5

$\underset{4\times5}{A^TC} + \underset{5\times4}{D}$

Cannot add - matrices must be the same size.

(c) $CD - 5A$

size of C: 4×5 $\checkmark$ size of D: 5×4 → 4×4

$\underset{4\times4}{CD} - \underset{4\times4}{5A}$ same size $\checkmark$

$\boxed{CD-5A \text{ is } 4\times4}$

(d) ~~$A^{-1}(CD)$~~^{T}

Typo: $A^T(CD)^T$

size of A^T: 4×4 $\checkmark$ size of $(CD)^T$: 4×4

$\boxed{A^T(CD)^T \text{ is } 4\times4}$

15. Niko has 7 white, 3 black, and 8 blue shirts. He randomly selects 6 shirts to take with him on a business trip. Let X represent the number of black shirts selected.

(a) Write the probability distribution for X.

Value of X	0	1	2	3
$P(X=x)$	$\frac{C(15,6)}{C(18,6)}$	$\frac{C(3,1)C(15,5)}{C(18,6)}$	$\frac{C(3,2)C(15,4)}{C(18,6)}$	$\frac{C(3,3)C(15,3)}{C(18,6)}$
	$\frac{55}{204}$	$\frac{33}{68}$	$\frac{15}{68}$	$\frac{5}{204}$

(b) How many black shirts can he expect to select?

$$E(X) = 0\left(\frac{55}{204}\right) + 1\left(\frac{33}{68}\right) + 2\left(\frac{15}{68}\right) + 3\left(\frac{5}{204}\right)$$

$$\boxed{E(X) = 1}$$

16. The odds in favor of the event A are 2:3. The odds in favor of the event B are 4:3. If A and B are mutually exclusive events, find the probability that A or B occurs.

$P(A \cap B) = 0$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= \frac{2}{2+3} + \frac{4}{4+3} - 0$$
$$= \frac{2}{5} + \frac{4}{7} = \boxed{\frac{34}{35}}$$

17. Several snack-sized bags of M&M's were examined to determine the number of M&M's in each package. The results of the study are given in the table below.

X

$L_2 \longrightarrow$ number of packages	5	0	4	2	7
Put X into $L_1 \longrightarrow X =$ number of M&M's	10	13	11	9	15

(a) What should the random variable X represent, number of packages or number of M&M's?

X = number of M&M's

(b) Give each of the following for this data set. Round to 4 decimal places when necessary.

i. E(X) = $\bar{x}$ in calc = 12.0556

ii. variance = 5.8302 (Mrs. Ramsey's class: Is this a sample or a population?)

sample so sample variance $= s_x^2 = 6.1732$

iii. mode = 15

iv. median = 11

18. Solve the following system of equations:

$$\begin{array}{rcrcrcr} x & - & 2y & + & z & = & -3 \\ 2x & + & y & - & 2z & = & 2 \\ -2x & + & 4y & - & 2z & = & 6 \end{array}$$

$$\left[\begin{array}{rrr|r} 1 & -2 & 1 & -3 \\ 2 & 1 & -2 & 2 \\ -2 & 4 & -2 & 6 \end{array}\right] \xrightarrow{\text{rref}} \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \left[\begin{array}{rrr|r} 1 & 0 & -3/5 & 1/5 \\ 0 & 1 & -4/5 & 8/5 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

Let $z = t$, where t is any real number.

$x - \frac{3}{5}z = \frac{1}{5}$

$x = \frac{3}{5}z + \frac{1}{5}$

$x = \frac{3}{5}t + \frac{1}{5}$

$y - \frac{4}{5}z = \frac{8}{5}$

$y = \frac{4}{5}z + \frac{8}{5}$

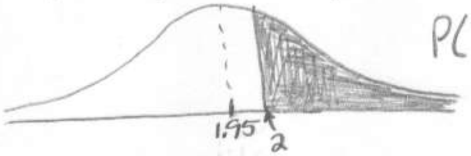
$y = \frac{4}{5}t + \frac{8}{5}$

$$\boxed{(x,y,z) = \left(\frac{3}{5}t + \frac{1}{5}, \frac{4}{5}t + \frac{8}{5}, t\right)}$$

infinitely many solutions.

19. The true volume of soda in 2-liter bottles packaged by Acme, Inc. is normally distributed with a mean of 1.95 L and a standard deviation of 0.35 L.

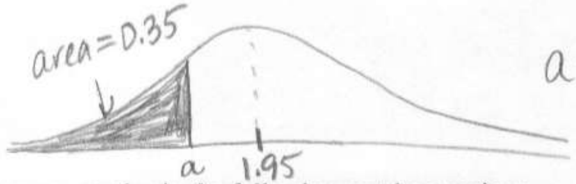
   (a) Find the probability that a randomly selected bottle contains more than 2 L of soda.



$P(X>2) = \text{normalcdf}(2, 1E99, \underset{\mu}{1.95}, \underset{\sigma}{0.35})$

$= \boxed{0.4432}$

   (b) 35% of all bottles contain a volume that is less than <u>1.8151</u> liters.



$a = \text{invNorm}(\underset{\text{area}}{0.35}, 1.95, \underset{\sigma}{0.35}) = 1.8151$

20. Solve for x, y, z, and w in the following matrix equation:

$$\underset{2\times 2}{\begin{bmatrix} x & 2z \\ 1 & 3 \end{bmatrix}} \underset{2\times 2}{\begin{bmatrix} 2 & y \\ -1 & 0 \end{bmatrix}} + \begin{bmatrix} -8 & w \\ 0 & -2 \end{bmatrix}^T = \begin{bmatrix} -4 & 12 \\ 5 & 10 \end{bmatrix}$$

$$\begin{bmatrix} 2x-2z & xy \\ 2-3 & y \end{bmatrix} + \begin{bmatrix} -8 & 0 \\ w & -2 \end{bmatrix} = \begin{bmatrix} -4 & 12 \\ 5 & 10 \end{bmatrix}$$

$$\begin{bmatrix} 2x-2z-8 & xy \\ -1+w & y-2 \end{bmatrix} = \begin{bmatrix} -4 & 12 \\ 5 & 10 \end{bmatrix}$$

$-1+w=5$

$\boxed{w=6}$

$y-2=10$

$\boxed{y=12}$

$xy=12$

$x(12)=12$

$\boxed{x=1}$

$2x-2z-8=-4$

$2(1)-2z=4$

$-2z=2$

$\boxed{z=-1}$

21. How many ways can 4 red crayons, 6 blue crayons, and 5 yellow crayons be <u>lined up in a row</u> if crayons of the same color are <u>identical</u>?

permutation

non-distinct (like MISSISSIPPI problem)

$$\frac{15!}{4!\,6!\,5!} = \boxed{630{,}630}$$

22. Elliot wants to buy Olivia 8 movies for her birthday. Olivia has given Elliot a list of 15 different movies that she would like to have, but Elliot left the list at home. How many ways can Elliot randomly choose 8 movies off a shelf that has 35 different movies (15 of which are on Olivia's list) and get at least 7 movies that are on the list?

7 or more

Exactly 7 or Exactly 8

$$C(15,7)C(20,1) + C(15,8) = \boxed{135,135}$$

23. A random variable X has a mean of 34 and a standard deviation of 2.5. Use Chebychev's inequality to estimate $P(28.75 \leq X \leq 39.25)$.

$34 - 28.75 = 5.25$ $\quad 39.25 - 34 = 5.25$

$5.25 = k * 2.5$

$2.1 = k$

$$P(28.75 \leq X \leq 39.25) \geq 1 - \frac{1}{2.1^2} = \boxed{\frac{341}{441}}$$

≈ 0.7732

24. Let $U = \{a,b,c,d,e,1,2,3,4,5\}$, and let $A = \{a,c,e,3,5\}$, $B = \{1,2,4,d\}$, and $C = \{c,e,3\}$. Which of the following are true?

$C^c = \{a,b,d,1,2,4,5\}$

(a) A and B are disjoint. True

(b) $\{a,2,5\} \subseteq B \cup C^c$ True $B \cup C^c = \{1,2,4,d,a,b,5\}$

(c) $e \subset C$ False

$\{e\} \subset C$ True or $e \in C$ True

(d) $\{c,e,3\} \in \{a,c,e,3,5\}$ False $\{c,e,3\} \subseteq \{a,c,e,3,5\}$ is true.

(e) A has 32 subsets. True $2^5 = 32$

(f) $(A \cap C)^c = \{3,5\}$ False

$A \cap C = \{c,e,3\}$

$(A \cap C)^c = \{a,b,d,1,2,4,5\}$

25. How much should Bob invest now in a savings account paying 2.25%/year compounded daily so that at the end of 10 years he has $25,000 in the account?

$N = 365 * 10$ $\quad PMT = 0$

$I\% = 2.25$ $\quad FV = 25000$

$PV = ?$ $\quad P/Y = C/Y = 365$

$\boxed{\text{Deposit } \$19,963.04}$

26. According to company records, 24% of all customers at Acme Hardware on any particular day buy caulk.

(a) Find the probability that among 35 randomly selected customers, at most 7 buy caulk that day.

$n = 35$
"success" – buy caulk
$p = .24$
Indep ✓

binomcdf(35, .24, 7) = $\boxed{0.3728}$

(b) How many of the 35 customers can be expected to buy caulk?

$$E(X) = np = 35 * 0.24 = \boxed{8.4}$$

27. (Section G.1) Roy and Clarice play a game in which they each flip a coin at the same time. If both coins land heads, Roy pays Clarice \$2. If both coins land tails, then Clarice pays Roy \$3. If Roy gets heads and Clarice gets tails, Clarice pays Roy \$5. Otherwise, Roy pays Clarice \$6.

(a) Write the payoff matrix for this two-person, zero-sum game.

(C) Clarice

$$\text{Roy}\quad \begin{array}{c} \\ H \\ T \\ \\ \end{array} \begin{array}{c} \begin{array}{cc} H & T \end{array} \\ \begin{bmatrix} -2\,mM & 5M \\ -6m & 3 \end{bmatrix} \\ \begin{array}{cc} -2 & 5 \end{array} \end{array} \begin{array}{l} \\ -2 \leftarrow R_1 \\ -6 \\ \\ \end{array}$$

(b) Is this game strictly determined? If yes, find the saddle point and state the value of the game. Also, state the optimal strategies for each player.

Yes. Saddle point = -2 in R1, C1.
value of game is -2.
Roy should play Row 1 and Clarice should play Column 1.

28. John borrowed \$378 from Bob, who charged him simple interest at a rate of 3.7% per year. After some time, John repaid Bob with \$401.31. How many months passed between the time that John borrowed the money and the time he repaid Bob?

$I = Prt$

$(401.31 - 378) = 378 * 0.037 * t$

$\frac{5}{3} = t$

$\frac{5}{3}$ years $* 12 \frac{\text{months}}{\text{year}} = \boxed{20 \text{ months}}$

29. If Ann is earning interest in a savings account at a rate of 11.2%/year compounded quarterly, by what percent will her account grow by in one year if she makes no additional deposits?

This is equivalent to asking for the effective annual yield (or effective rate of interest).

Eff(11.2, 4) $\approx \boxed{11.6792\%}$

30. A business analyst studied a group of car buyers in a metropolitan area (all with gross annual incomes of $1 million or more) and found that of those who currently own an economy car, 25% will buy a mid-sized sedan and 5% will buy a luxury sedan for their next car. Of those who currently own a mid-sized sedan, 40% will buy an economy car and 15% will buy a luxury sedan for their next car. All of those who currently own a luxury sedan will never buy any other type of car again.

(a) Write the transition matrix for this Markov process. If there are any absorbing states, list them first in the matrix.

E - economy car
m - mid-sized sedan
L - Luxury sedan

Current

$$T = \text{Next}\ \begin{array}{c} \\ L \\ E \\ m \end{array} \begin{array}{c} \begin{array}{ccc} L & E & m \end{array} \\ \left[\begin{array}{c|cc} 1 & 0.05 & 0.15 \\ \hline 0 & 0.7 & 0.4 \\ 0 & 0.25 & 0.45 \end{array}\right] \end{array}$$

L is an absorbing state.

(b) If this transition matrix is for a regular Markov process, find the steady-state distribution. If this transition matrix is for an absorbing Markov process, find the limiting matrix.

$$\text{Limiting Matrix} = \begin{array}{c} \\ L \\ E \\ m \end{array} \begin{array}{c} \begin{array}{ccc} L & E & m \end{array} \\ \left[\begin{array}{c|cc} 1 & 1 & 1 \\ \hline 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right] \end{array}$$

$A(I_2-B)^{-1}$ where $A = [0.05\ \ 0.15]$ and $B = \begin{bmatrix} 0.7 & 0.4 \\ 0.25 & 0.45 \end{bmatrix}$

(c) What is the probability that someone who initially owned a mid-sized sedan will eventually own a luxury sedan?

Probab. is 1.

31. (Section G.2) Let $A = \begin{bmatrix} 4\,\text{M} & 2\,\text{m} \\ -5\,\text{m} & 6\,\text{M} \end{bmatrix}$ be the payoff matrix for a two-person, zero-sum game.

not strictly determined

(a) Find the optimal mixed strategies for each player.

$P = [p_1\ \ p_2]$ $\quad p_1 = \dfrac{d-c}{a+d-b-c} = \dfrac{6+5}{4+6-2+5} = \dfrac{11}{13}$ so $p_2 = 1 - \frac{11}{13} = \frac{2}{13}$

Row player's optimal mixed strategy is $P = [\frac{11}{13}\ \ \frac{2}{13}]$.

$Q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$ $\quad q_1 = \dfrac{d-b}{a+d-b-c} = \dfrac{6-2}{4+6-2+5} = \dfrac{4}{13}$ so $q_2 = 1 - \frac{4}{13} = \frac{9}{13}$

Column player's optimal mixed strategy is $Q = \begin{bmatrix} 4/13 \\ 9/13 \end{bmatrix}$

(b) Find the expected payoff to the row player if both players use their optimal mixed strategies.

$$E = PAQ = [\tfrac{11}{13}\ \ \tfrac{2}{13}] \begin{bmatrix} 4 & 2 \\ -5 & 6 \end{bmatrix} \begin{bmatrix} 4/13 \\ 9/13 \end{bmatrix} = [2.62]$$

The row player can expect to win \$2.62.

Another method:

$$E = \frac{ad-bc}{a+d-b-c} = \frac{4 \cdot 6 - 2(-5)}{4+6-2+5} = \frac{34}{13} \approx 2.62$$