Math 142 - Final Exam Review

NOTE: This review is intended to highlight the material covered on the Final Exam but should not be used as your sole source of practice. Also refer to your instructor's lecture notes, previous week-in-reviews, previous exams and quizzes, suggested homework, supplemental homework, and the online homework as additional sources for review and exam preparation.

1. Find the locations of all local extrema and saddle points of each of the following functions.

   (a) $f(x,y) = -0.5x^2 + 5xy - 2y^2 + 10x + 13y + 1$

   (b) $g(x,y) = 2x^2 - 2x^2y + 6y^3$

2. A rectangular box is to have a square base and a volume of 20 cubic feet. If the material for the base costs 30 cents per square foot, the material for the sides costs 10 cents per square foot, and the material for the top costs 20 cents per square foot, determine the dimensions of the box that can be constructed at minimum cost. (source: #8, pg. 365 of *Applied Calculus for the Managerial, Life, and Social Sciences*, 5th ed., by Tan)

3. What sequence of graph transformations must be performed to obtain the graph of $h(x) = -5\mid x+1\mid -6$ from the graph of $f(x) = \mid x\mid$?

4. Find the instantaneous rate of change of $g(x) = \ln(7x-5)$ at $x = 2$.

5. The table below shows the population of a city in China for different years.

Year	1930	1940	1950	1960	1970	1980
Population (in thousands)	390	700	900	1000	950	830

   (a) Find the best model $P(t)$ for the data, assuming that the model will be used for interpolation purposes only. Let t represent the number of years since 1930.

   (b) Use your model to estimate when the population of the city reached 750,000 people.

6. Compute each of the following by hand.

   (a) $\displaystyle\int_1^a (7x^{-1} + x^{0.2})dx$ where $a > 1$

   (b) $\displaystyle\int_9^{21} (t+1)\sqrt{t-5}\,dt$

7. Find the domain of each of the following functions.

   (a) $f(x) = \dfrac{\ln(x-7)}{\sqrt[3]{x^2-16}}$

   (b) $g(x) = \begin{cases} \sqrt[8]{10-x} & \text{if } x \le 20 \\ \dfrac{1}{x^3 - 38x^2 - 80x} & \text{if } x > 20 \end{cases}$

8. Bob purchased a tractor for \$27,500 in 1980 and 7 years later, it was worth only \$14,788. Assuming linear depreciation, find when the tractor will be worth \$5,700.

9. Find the equation of $g(x)$ if its graph is obtained by taking the graph of $f(x) = e^x$ and shifting it 8 units to the right, stretching it vertically by a factor of 7, and then shifting it down 2 units.

10. C&G Imports, Inc., imports two brands of white wine, one from Germany and the other from Italy. The German wine costs \$4 per bottle, and the Italian wine can be obtained for \$3 per bottle. It has been estimated that if the German wine retails at p dollars per bottle and the Italian wine is sold for q dollars per bottle, then $x = 2000 - 150p + 100q$ bottles of German wine and $y = 1000 + 80p - 120q$ bottles of Italian wine will be sold per week. (source: #24, pg. 635 of *Applied Calculus for the Managerial, Life, and Social Sciences*, 5th ed., by Tan)

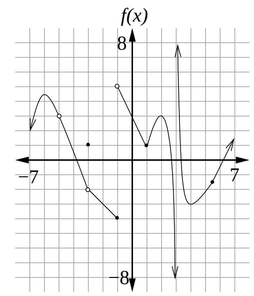
    (a) Find the weekly revenue function $R(p,q)$.

    (b) Find the weekly profit function $P(p,q)$.

    (c) Evaluate and interpret $P_q(15,12)$

    (d) Determine the unit price for each brand that will allow C&G to realize the largest possible weekly profit.

11. Refer to the graph of $f(x)$ to find each of the following:



    (a) $\lim_{x\to 6^+} f(x)$

    (b) $\lim_{x\to -1^+} f(x)$

    (c) $\lim_{x\to 3^-} f(x)$

    (d) $\lim_{x\to -3} f(x)$

    (e) Find all x for which $f(x)$ is discontinuous on the interval $(-7,5)$.

12. Find the equation of the tangent line to $f(x) = 0.5e^{2x}$ at $x = 1$.

13. Find the absolute extrema of $f(x) = xe^{0.2x}$ on the interval $[-6,1]$.

14. Find each of the following limits, if they exist.

    (a) $\lim_{x\to -3} \dfrac{x^2+8x+15}{x^2+x-6}$

    (b) $\lim_{x\to 0} \dfrac{2x^2-7}{5x+14}$

    (c) $\lim_{x\to \infty} \dfrac{4x^2-3x^3+2}{5-7x+12x^2}$

    (d) $\lim_{x\to -\infty} \dfrac{7x^4+5x}{10x^3-21x^4}$

15. Find all asymptotes and x-coordinates of any holes for $f(x) = \dfrac{5x^2-10x-15}{(x+2)^2(x^2-3x)}$

16. Find $\lim_{x\to -\infty} (-6x^3+3x-7)$.

17. Bob runs a tutoring service. He prices each tutoring session as follows: For a group of 20 or more people, he charges the group a flat fee of \$200 plus \$10 per person. For a group of 8 to 19 people, he charges the group a flat fee of \$125 plus \$15 per person. For a session with fewer than 8 people, he charges a flat fee of \$55 plus \$20 per person. Write a piecewise-defined function to represent the total bill for one session with a group of x people.

18. Use the definition of the derivative to find $f'(x)$ for $f(x) = x - 9x^2$.

19. Acme Music Company has determined the price-demand function for its home audio system to be $p = 3080 - 10x$ dollars per system. Acme Music has fixed costs that amount to \$56,250 and variable costs of \$580 per system.

    (a) How many home audio systems must be sold to maximize revenue? Use two methods to obtain your answer.

    (b) Find the profit and marginal profit from the production and sale of 175 home audio systems.

(c) Use your answers in (b) to approximate the profit from the production and sale of 177 home audio systems.

(d) How many home audio systems should be produced and sold so that Acme breaks even?

(e) Find the marginal average cost when 100 home audio systems are produced.

20. Find the derivative of each of the following functions. Simplify the function before taking the derivative where possible.

(a) $h(x) = e^{-4.5\ln(x^2+1)}$

(b) $f(x) = 3x^2 - \dfrac{5}{\sqrt[4]{x}} + e^{-x} + 7^{2x} - e^{-2}$

(c) $g(x) = \log_8\left((2x-7)\sqrt{x+3}\right)$

(d) $g(x) = (10x^3 - 1)e^{\sqrt{x}}$

(e) $m(t) = \left(3\sqrt{t^2+1} - \dfrac{5}{t}\right)^8$

(f) $h(x) = \dfrac{\ln x + 4x^7}{2x+9}$

(g) $f(x) = \dfrac{\left(2\cdot 3^{-5x}\right)^4}{27^{x+2}}$

21. Solve each of the following for x or b as indicated.

(a) $\log_b 64 = \dfrac{3}{2}$

(b) $\log_9 x = -3$

(c) $5^{x^2-4x} = \dfrac{1}{25^x}$

(d) $3^x \cdot 8x + 3^{x+1} = 0$

(e) $\log_2(x+2) + \log_2 x = 2\log_2 3 + \log_2 10 - \log_2 6$

22. In 1965, Bob invested \$300 in a savings account paying 4.2% per year compounded continuously.

(a) What was the average rate of change of this account's value from 1972 to 1975?

(b) What was the average value (i.e., average balance) of this account from 1972 to 1975?

23. Find the intervals where $f(x) = 3e^{2x} - 8e^x$ is concave upward and concave downward, and find the coordinates of any inflection points.

24. Find the area between $f(x) = x+4$ and $g(x) = (x+2)^2$ on the interval $[-2, 1]$.

25. The management of Bob's Auto Shop has determined the price-demand equation for its air filters to be $2x + 100p = 2000$ where x is the number of filters sold when the unit price is $\$p$.

(a) Find the elasticity of demand $E(p)$.

(b) Find and interpret $E(9)$.

(c) If the current price of \$9 per air filter is increased by 3%, what would be the approximate change in demand?

(d) If the current price per air filter is \$13, should Bob increase or decrease this price to produce an increase in revenue?

26. A company's marginal profit can be modeled by $P'(x) = 18\sqrt{x+5}$ dollars per item, where x is the number of items produced and sold. If this company's profit is \$11,475 when 95 items are produced and sold, find $P(x)$.

27. On a particular day during monsoon season, rain was falling at a rate given by $r(t) = \frac{5}{t+2}$ cm per hour, where t is the number of hours since 3am, $0 \le t \le 21$.

    (a) Approximate the area under $r(t)$ from $t = 2$ to $t = 3$ using a left sum with 3 rectangles. What does this area represent?

    (b) Find the exact amount of rain that fell from 8am to 2pm this day.

28. Find the intervals where $f(x) = \frac{8}{x} + x$ is increasing and decreasing, and find the coordinates of any local extrema.

29. Compute each of the following.

    (a) $\int \left(4e^x - 7x^3 + 4x^{-1} - e^2\right) dx$

    (b) $\int 4t^3\sqrt{t^4 - 7}dt$

    (c) $\int \frac{x^6}{4x^7 - 5}dx$

    (d) $\int (4x+2)e^{x^2+x}dx$

    (e) $\int \frac{2}{x\sqrt{5+\ln x}}dx$

30. Let $f(x,y) = \frac{7x^2y^4}{x^3y - 7y}$.

    (a) Find $f(2,10)$.

    (b) Find the first-order partial derivatives of $f(x,y)$.

31. Let $Q(x)$ be an antiderivative of $q(x)$. Find $Q(3)$ if $\int_3^7 q(x)dx = -9$ and $Q(7) = 5$.

32. Mariska is selling lemonade. When she charges \$5 per glass, she can sell 36 glasses of lemonade. For every \$0.10 that she lowers the price, an additional 4 glasses of lemonade will be sold. What price should Mariska charge per glass to maximize revenue?

33. Find the producers' surplus at equilibrium price level for a product whose price-demand equation is given by $p = D(x) = 430 - 0.62x$ dollars per item and price-supply equation is given by $p = S(x) = 0.004x^2 + 25$ dollars per item.

34. The productivity of an automobile parts manufacturing company is given by $f(x,y) = 30x^{0.35}y^{0.65}$ units, where x and y represent the number of units of labor and capital utilized, respectively.

    (a) Find $f_x(x,y)$ and $f_y(x,y)$.

    (b) Find the marginal productivity of labor when 70 units of labor and 50 units of capital are utilized.

    (c) Find the marginal productivity of capital when 70 units of labor and 50 units of capital are utilized.

    (d) For the greatest increase in productivity, should this company increase the use of labor or capital (assuming they are currently using 70 units of labor and 50 units of capital)?

35. Find all second-order partial derivatives of $f(x,y) = 2xe^{xy}$.