Math 166 - Final Exam Review

The following review covers Sections 2.1-2.7, 3.1-3.3, 5.1-5.3, A.1, A.2, 6.1-6.4, 7.1-7.6, 8.1-8.6, 9.1, 9.2, and 9.4 and contains about 1 problem (or 1 part of a problem) per section. Since many sections contain multiple topics that can be addressed in separate problems, this review should not be used as your sole source of practice problems as you study for the final exam. You should also review all examples demonstrated in lecture, the suggested homework problems from your textbook, the problems from the online homeworks, and problems from previous week-in-reviews.

1. Acme, Inc. has two best-selling products, Product A and Product B. Making one unit of Product A requires 40 grams of material and 5 minutes of labor, and making one unit of Product B requires 20 grams of material and 15 minutes of labor. Acme, Inc. has a total of 3,600 grams of material available and has enough employees to work a combined total of 25 hours. Since manufacturing these two products produces a lot of radioactive waste, the local government has mandated that Acme's combined production of these two products cannot exceed 120 units.

   (a) If the profit per unit for Product A is \$1.00, and the profit per unit for Product B is \$1.20, how many of each type of product should be produced in order to maximize profit?

Let x = the number of units of Product A and y = the number of units of Product B.

	Product A	Product B	Amt. Avail.
material	40g	20g	3,600 g
labor	5 min	15 min	25 hrs (1500 min)

Other constraint: $x+y \le 120$

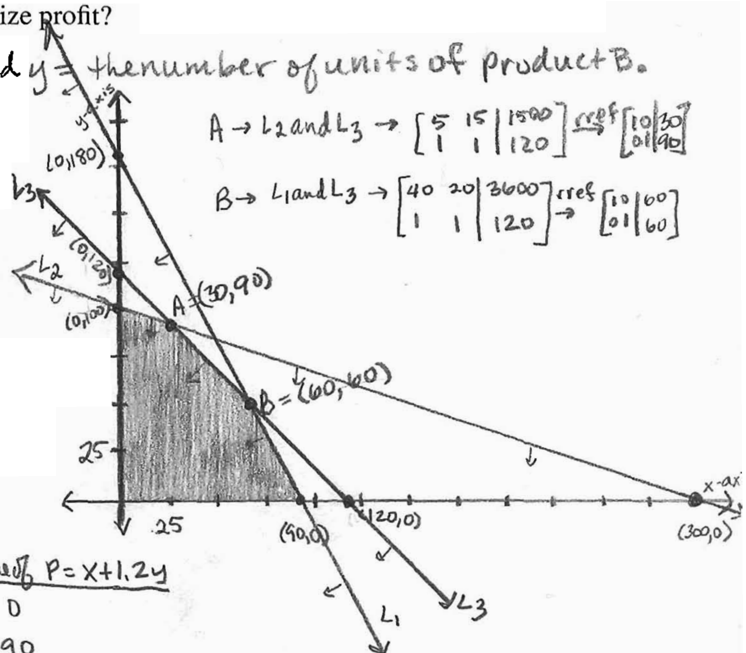
Maximize Profit $P = x + 1.2y$

subject to
$$40x + 20y \le 3600$$
$$5x + 15y \le 1500$$
$$x + y \le 120$$
$$x \ge 0,\ y \ge 0$$

L_1	L_2	L_3
$40x+20y=3600$	$5x+15y=1500$	$x+y=120$
(0,180), (90,0)	(0,100) (300,0)	(0,120), (120,0)
Test (0,0)	Test (0,0)	Test (0,0)
$0 \le 3600$ ✓	$0 \le 1500$ ✓	$0 \le 120$ ✓

A → L_2 and L_3 → $\left[\begin{array}{cc|c} 5 & 15 & 1500 \\ 1 & 1 & 120 \end{array}\right] \xrightarrow{rref} \left[\begin{array}{cc|c} 1 & 0 & 30 \\ 0 & 1 & 90 \end{array}\right]$

B → L_1 and L_3 → $\left[\begin{array}{cc|c} 40 & 20 & 3600 \\ 1 & 1 & 120 \end{array}\right] \xrightarrow{rref} \left[\begin{array}{cc|c} 1 & 0 & 60 \\ 0 & 1 & 60 \end{array}\right]$



Corner	Value of $P = x + 1.2y$
(0,0)	0
(90,0)	90
(60,60)	132
(30,90)	138 ←
(0,100)	120

The maximum profit is \$138 and occurs when 30 of product A and 90 of Product B are produced.

   (b) If the profit per unit for Product A is \$2.00, and the profit per unit for Product B is \$2.00, how many of each type of product should be produced in order to maximize profit?

Maximize $P = 2x + 2y$

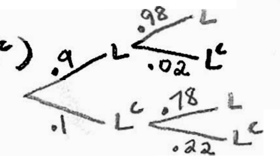
Corner	Value of $P = 2x+2y$
(0,0)	0
(90,0)	180
(60,60)	240 ←
(30,90)	240 ←
(0,100)	200

The maximum value of P is \$240 and occurs when t units of Product A and $120-t$ units of Product B are produced, where $t = 30, 31, 32, \ldots 60$.

2. An insurance company classifies drivers as low-risk if they are accident-free for 1 year. Past records indicate that 98% of the drivers in the low-risk category one year will remain in that category the next year, and 78% of the drivers who are not in the low-risk category one year will move to the low-risk category the next year. (pg. 598 of *Finite Mathematics for Business, Economics, Life Sciences, and Social Sciences* by Barnett, Ziegler, and Byleen)

   (a) If 90% of the drivers in the community are in the low-risk category this year, what is the probability that a driver chosen at random from the community will be in the low-risk category two years from now?

State1 = Low Risk (L)
State2 = not Low Risk (L^c)



$T=\begin{bmatrix} .98 & .78 \\ .02 & .22 \end{bmatrix}$, $X_0=\begin{bmatrix} .9 \\ .1 \end{bmatrix}$ $\quad X_2=T^2X_0=\begin{bmatrix} .972 \\ .028 \end{bmatrix}$

Probability of Low-risk 2 years from now $=\boxed{.972}$

   (b) If this trend continues, what percent of the drivers in the community will be in the low-risk category in the long run? ← steady state

$TX=X$

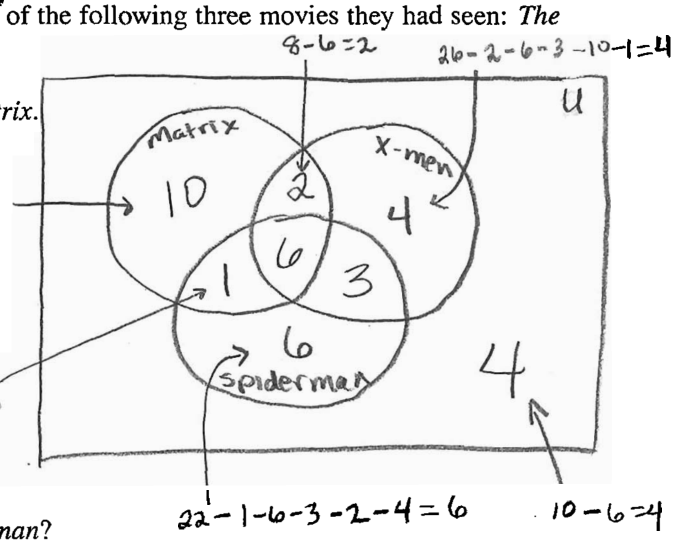
$\begin{bmatrix} .98 & .78 \\ .02 & .22 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}=\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ $\quad \begin{matrix} .98x_1+.78x_2=x_1 \\ .02x_1+.22x_2=x_2 \end{matrix} \rightarrow \begin{matrix} -.02x_1+.78x_2=0 \\ .02x_1-.78x_2=0 \\ x_1+x_2=1 \end{matrix}$ $\quad \left[\begin{array}{cc|c} -.02 & .78 & 0 \\ .02 & -.78 & 0 \\ 1 & 1 & 1 \end{array}\right] \xrightarrow{rref} \left[\begin{array}{cc|c} 1 & 0 & .975 \\ 0 & 1 & .025 \\ 0 & 0 & 0 \end{array}\right]$

In the long run, $\boxed{97.5\%}$ of the drivers in the community will be in the low-risk category.

3. A survey of some college students was conducted to see which of the following three movies they had seen: *The Matrix*, *X-Men*, and *Spiderman*. It was found that

(order of use)

- (1) 3 students had seen *X-Men* and *Spiderman* but not *The Matrix*.
- (6) 26 students had seen *The Matrix* or *X-Men*.
- (4) 6 students had seen exactly 2 of the 3 movies.
- (7) 22 students had seen *X-Men* or *Spiderman*.
- (8) 10 students had seen neither *X-Men* nor *The Matrix*.
- (2) 6 students had seen all three movies.
- (5) 19 students had seen *The Matrix*.
- (3) 8 students had seen *The Matrix* and *X-Men*.



   (a) How many students were surveyed?

$10+2+4+1+6+3+6+4=\boxed{36}$

   (b) How many students surveyed had seen *X-Men* and *Spiderman*?

$6+3=\boxed{9}$

   (c) How many students surveyed had seen exactly one of these three movies?

$10+4+6=\boxed{20}$

   (d) What is the probability that a randomly selected student from this survey had seen none of the three movies?

$\frac{4}{36}=\boxed{\frac{1}{9}}$

   (e) What is the probability that a randomly selected student from this survey had seen *The Matrix* or *Spiderman* but not both?

$\frac{10+2+6+3}{36}=\frac{21}{36}=\boxed{\frac{7}{12}}$

4. Amelia wants to save enough money so that she will have \$3,000 to spend on a trip to Europe that she is planning to take in 5 years. If she opens an account paying 6% interest compounded monthly with \$400 and makes monthly deposits for 5 years, what is the size of the monthly payment that will reach her goal?

$N = 5*12$ $PMT = ?$ → $\boxed{\$35.27}$

$I\% = 6$ $FV = 3000$

$PV = -400$ $P/Y = C/Y = 12$

5. In an experiment, several people are randomly surveyed to find out if they consider themselves to be a Republican, Democrat, or Independent. Each person's response, along with his or her sex, is recorded.

   (a) Write an appropriate sample space for this experiment.

   M - Male D - Democrat
   F - Female I - Independent
   R - Republican

   $S = \{(M,R),(M,D),(M,I),(F,R),(F,D),(F,I)\}$

   (b) Write the event that the person surveyed is female.

   $E = \{(F,R),(F,D),(F,I)\}$

   (c) Write the event that the person surveyed considers himself or herself to be an Independent.

   $F = \{(M,I),(F,I)\}$

   (d) Are the events found in (b) and (c) mutually exclusive?

   Mutually exclusive means no outcomes in common.
   Since E and F have the outcome (F, I) in common, i.e., $E \cap F = \{(F,I)\}$, E and F are NOT mutually exclusive.

6. Consider the propositions

   p: Some of the presents are wrapped.
   q: All of the guests have arrived.
   r: The food is not ready.

   (a) Write symbolically the statement, "All of the guests have arrived, but the food is not ready and none of the presents are wrapped."

   $q \wedge r \wedge \sim p$

   (b) Write symbolically the statement, "Either the food is ready or all of the guests have arrived, but not both."

   $\sim r \veebar q$

   exclusive disjunction

   (c) Write the statement $(\sim q \vee r) \wedge p$ in English.

   Not all of the guests have arrived or the food is not ready, but some of the presents are wrapped.

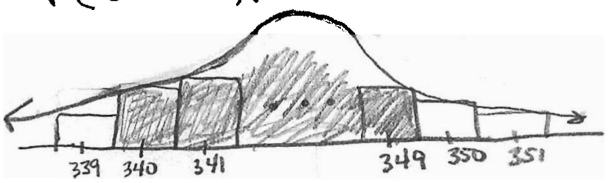
   (d) Write the statement $\sim (r \vee p)$ in English.

   $\sim(r \vee p) = \sim r \wedge \sim p$

   The food is ready and none of the presents are wrapped.

7. The Capital Two credit card company has found that 23% of its cardholders are late in making their monthly payments. Using the normal approximation to the binomial distribution, find the probability that among 1,500 randomly selected cardholders, at least 340 but fewer than 350 are late in making their payment in a particular month. X = # of card holders that are late in making their payment

$P(340 \le X < 350) \approx \text{normalcdf}(339.5, 349.5, 345, \sqrt{265.65}) = \boxed{.2409}$



$\mu = np = 1500 * .23 = 345$

$\sigma = \sqrt{npq} = \sqrt{1500 * .23 * (1 - .23)}$

$= \sqrt{265.65}$

8. An economy consists of two industries: manufacturing and transportation. The production of one unit of manufacturing goods requires the consumption of 0.3 units of manufacturing goods and 0.2 units of transportation goods. The production of one unit of transportation goods requires the consumption of 0.5 units of manufacturing goods and 0.4 units of transportation goods. m = manufacturing goods
t = transportation goods

(a) How many units of transportation goods are consumed in the production of 2,500 units of manufacturing goods?

Input-output matrix

$$A = \begin{array}{cc} & \text{output} \\ & \begin{array}{cc} m & t \end{array} \\ \text{input} \begin{array}{c} m \\ t \end{array} & \begin{bmatrix} .3 & .5 \\ .2 & .4 \end{bmatrix} \end{array}$$

To output 2500 units of m,

$(.2)(2500) = \boxed{500 \text{ units of } t}$ are consumed.

(b) How many units of manufacturing goods are required to produce 500 units of each of the two types of goods in this economy?

$.3(500) + .5(500) = \boxed{400 \text{ units of } m}$

(c) What is the total production required to meet an external demand of 3,500 units of manufacturing goods and 6,300 units of transportation goods?

$D = \begin{bmatrix} 3500 \\ 6300 \end{bmatrix}$

X = total production $= \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ where x_1 = total # of units of m
x_2 = total # of units of t

$X = (I_2 - A)^{-1} D = \begin{bmatrix} 16406.25 \\ 15968.75 \end{bmatrix}$ $\boxed{\begin{array}{l} 16{,}406.25 \text{ units of } m \\ 15{,}968.75 \text{ units of } t \end{array}}$

(d) How many units of each type of good are consumed internally in meeting the demand given in part (c)?

Internal consumption: either AX or $X - D$ (both give same answer)

$AX = X - D = \begin{bmatrix} 12906.25 \\ 9668.75 \end{bmatrix}$ $\boxed{\text{12,906.25 units of m and 9,668.75 units of t are consumed internally.}}$

Reword:
The # of years Bob has been alive is 3 times the # of yrs Sue has been alive.
$$\underbrace{\text{The \# of years Bob has been alive}}_{y} = 3 * \underbrace{\text{\# of yrs Sue has been alive}}_{z}$$

9. Fred, Bob, and Sue are all aliens with extremely long life expectancies. Bob and Sue's combined age in years is only half that of Fred's, and Bob has been alive for three times as many years as Sue. If the three friends' combined ages total 504 years, how old is each alien?

Let x = Fred's age in years.
Let y = Bob's age in years.
Let z = Sue's age in years.

$$y + z = \frac{1}{2}x \qquad -\frac{1}{2}x + y + z = 0$$
$$y = 3z \longrightarrow y - 3z = 0$$
$$x + y + z = 504 \qquad x + y + z = 504$$

$$\begin{array}{c} \begin{matrix} x & y & z & \end{matrix} \\ \left[\begin{array}{ccc|c} -\frac{1}{2} & 1 & 1 & 0 \\ 0 & 1 & -3 & 0 \\ 1 & 1 & 1 & 504 \end{array}\right] \end{array} \xrightarrow{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 336 \\ 0 & 1 & 0 & 126 \\ 0 & 0 & 1 & 42 \end{array}\right]$$

$\boxed{\text{Fred is 336 years old. Bob is 126 years old. Sue is 42 years old.}}$

10. Six people get into an elevator that services 12 floors in a certain building. What is the probability that these six people exit the elevator with at least 2 people exiting on the same floor?

E = event that at least 2 exit on the same floor.
E^c = event that they all exit on different floors.

$$P(E) = 1 - P(E^c) = 1 - \frac{n(E^c)}{n(S)} = 1 - \frac{P(12,6)}{12^6} = \boxed{\frac{1343}{1728}}$$

$n(S)$: $\underset{1st}{12}\ \underset{2nd}{12}\ \underset{3rd}{12}\ \underset{4th}{12}\ \underset{5th}{12}\ \underset{6th}{12} = 12^6$ person

$n(E^c) = \underset{1st}{12} \cdot \underset{2nd}{11} \cdot \underset{3rd}{10} \cdot \underset{4th}{9} \cdot \underset{5th}{8} \cdot \underset{6th}{7} = P(12,6)$

11. Construct the truth table for $\sim(p\wedge \sim q) \vee p$.

p	q	$\sim q$	$p\wedge\sim q$	$\sim(p\wedge\sim q)$	$\sim(p\wedge\sim q)\vee p$
T	T	F	F	T	T
T	F	T	T	F	T
F	T	F	F	T	T
F	F	T	F	T	T

12. Perform the next pivot in the Gauss-Jordan method:

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 3 \\ 0 & 5 & 6 & 0 \\ 0 & -4 & 3 & -1 \end{array}\right] \xrightarrow{R_2+R_3\to R_2} \left[\begin{array}{ccc|c} 1 & 0 & 2 & 3 \\ 0 & 1 & 9 & -1 \\ 0 & -4 & 3 & -1 \end{array}\right] \xrightarrow{R_3+4R_2\to R_3} \left[\begin{array}{ccc|c} 1 & 0 & 2 & 3 \\ 0 & 1 & 9 & -1 \\ 0 & 0 & 39 & -5 \end{array}\right]$$

$$\overset{R_2}{[0\ 5\ 6\ 0]} \overset{+}{+} \overset{R_3}{[0\ -4\ 3\ -1]}$$
$$= [0\ 1\ 9\ -1] \to R_2$$

$$\overset{R_3}{[0\ -4\ 3\ -1]} \overset{+}{+} \overset{4R_2}{4[0\ 1\ 9\ -1]}$$
$$+\ [0\ 4\ 36\ -4]$$
$$[0\ 0\ 39\ -5] \to R_3$$

* This is not the only correct way to work this problem.

13. Classify each of the following types of random variables as either finite discrete, infinite discrete, or continuous, and state the possible values of the random variable.

(a) $X =$ the number of black cats being boarded at a kennel that can house at most 45 animals.

finite discrete $X = 0, 1, 2, 3, \ldots, 45$

(b) $Y =$ the number of gallons of water in a bucket that has a maximum capacity of 5 gallons.

continuous $0 \le Y \le 5$

(c) $Z =$ the number of times my phone rings before I answer it.

infinite discrete $X = 1, 2, 3, 4, \ldots$

14. Bob just bought a house for \$200,000. He made a 15% down payment and financed the balance with a loan that has an interest rate of 5.45%/year compounded monthly.

85% of 200000

$.85(200000) = 170000$

(Down payment is \$30,000)

(a) If the loan is ammortized for 25 years, what is Bob's monthly payment?

$N = 25 * 12$ $PMT = ? \rightarrow$ $\boxed{\$1,038.88}$

$I\% = 5.45$ $FV = 0$

$PV = 170000$ $P/Y = C/Y = 12$

(b) After 10 years of payments, how much equity has Bob earned on his house?

Equity = Value of house − what is still owed

$N = 10 * 12$ $PMT = -1038.88$

$I\% = 5.45$ $FV = ? \rightarrow \$127,558.37$

$PV = 170000$ $P/Y = C/Y = 12$

$= 200,000 - 127558.37$

$= \boxed{\$72441.63}$

(c) How much interest will Bob pay on the loan?

Interest paid = Total amt paid for house − original cost of house

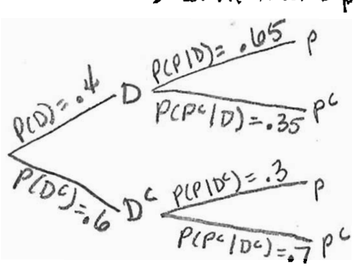
$= 30,000 + 25 * 12 * 1038.88 - 200,000$

$= 341,664 - 200000 = \boxed{\$141,664}$

15. Forty percent of all patrons at a local movie theater buy a drink, and of those who buy a drink, 65% also buy popcorn. Only 30% of those who do not buy a drink buy popcorn.

(a) What is the probability that a patron who did not buy popcorn (given) did buy a drink?

D-event that a patron buys a drink ; P = event that a patron buys popcorn.



$$P(D \mid P^c) = \frac{P(D \cap P^c)}{P(P^c)} = \frac{(.4)(.35)}{(.4)(.35) + (.6)(.7)}$$

$$= \boxed{.25}$$

(b) What is the probability that a patron did buy a drink but did not buy popcorn?

and → intersection

$P(D \cap P^c) = (.4)(.35) = \boxed{.14}$

(c) Are the events of buying a drink and buying popcorn independent? Why or why not?

$P(D) = .4$ (on tree)

$P(P) = (.4)(.65) + (.6)(.3) = .44$

(By circling the P's, multiplying, then adding

$P(D \cap P) = (.4)(.65) = .26$

(By multiplying through the branches.

Test for Indep.

$P(D \cap P) \stackrel{?}{=} P(D)P(P)$

$.26 \stackrel{?}{=} (.4)(.44)$

$.26 \stackrel{?}{=} .176$ NO!

$\boxed{\text{Not indep. since } P(D \cap P) \neq P(D)P(P).}$

16. If A is a 4×4 singular matrix, B is a 2×4 matrix, C is a 4×5 matrix, and D is a 5×4 matrix, which of the following are possible? If the operation IS possible, give the size of the resulting matrix. If the operation is NOT possible, explain why.

(a) BA

$$\underbrace{\underset{2\times4}{B} \quad \underset{4\times4}{A}}_{2\times4}$$

$\boxed{BA \text{ is } 2\times4.}$

(b) A^TC+D

$$\underbrace{\underset{4\times4}{A^T} \quad \underset{4\times5}{C}}_{4\times5}$$

$\underset{4\times5}{A^TC} + \underset{5\times4}{D}$

$\boxed{\text{Not possible: Matrices must be the same size to add.}}$

(c) $CD-5A$

$$\underbrace{\underset{4\times5}{C} \quad \underset{5\times4}{D}}_{4\times4}$$

$\underset{4\times4}{CD} - \underset{4\times4}{5A}$

$\boxed{CD-5A \text{ is } 4\times4}$

(d) $A^{-1}(CD)^T$

Not possible: A is singular, which means it has no inverse.

17. A drawer contains 5 white socks and 7 black socks. (Assume that all white socks coordinate with each other, and all black socks coordinate with each other.) A sample of 4 socks is drawn (without replacement) from the drawer. How many matching pairs of socks can you expect to have in the sample?

Let X = the # of matching pairs of socks in a sample of 4.

what can happen in a sample of 4 socks:
- 4 white, 0 black ← 2 matching pairs
- 3 white, 1 black ← 1 matching pair
- 2 white, 2 black ← 2 matching pairs
- 1 white, 3 black ← 1 matching pair
- 0 white, 4 black ← 2 matching pair

Value of X	P(X=x)
1	49/99
2	50/99

$P(X=1) = P(3W1B \text{ or } 1W3B) = \dfrac{C(5,3)C(7,1)+C(5,1)C(7,3)}{C(12,4)} = \dfrac{49}{99}$

$P(X=2) = 1-\dfrac{49}{99} = \dfrac{50}{99}$ $\quad E(X) = 1\left(\dfrac{49}{99}\right)+2\left(\dfrac{50}{99}\right) = \dfrac{149}{99} \approx \boxed{1.5051}$

18. The odds in favor of the event A are 2:3. The odds in favor of the event B are 4:3. If A and B are mutually exclusive events, find the probability that A or B occurs.

$P(A) = \frac{2}{2+3} = \frac{2}{5}$

$P(B) = \frac{4}{4+3} = \frac{4}{7}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{2}{5} + \frac{4}{7} - 0$ ← since A and B are mutually exclusive.

$= \boxed{\frac{34}{35}}$

19. Several snack-sized bags of M&M's were examined to determine the number of M&M's in each package. The results of the study are given in the table below.

$= X$

number of packages	5	0	4	2	7 ← L_2
$X =$ number of M&M's	10	13	11	9	15 ← L_1

1VarStats L_1, L_2

(a) What should the random variable X represent, number of packages or number of M&M's?

X = number of M&M's

(b) Give each of the following for this data set. Round to 4 decimal places when necessary.

i. E(X) = 12.0556

ii. (sample) variance = $s_x^2 = 6.1732$ ← NOTE: If your instructor never mentioned the difference between sample statistics and population statistics, your answer is $\sigma_x^2 = 5.8302$.

iii. mode = 15

iv. median = 11

20. Solve the following system of equations:

$$\begin{aligned} x - 2y + z &= -3 \\ 2x + y - 2z &= 2 \\ -2x + 4y - 2z &= 6 \end{aligned}$$

$$\begin{array}{cccc} x & y & z & \\ \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & -2 & 1 & -3 \\ 2 & 1 & -2 & 2 \\ -2 & 4 & -2 & 6 \end{array}\right] \xrightarrow{rref} \left[\begin{array}{ccc|c} 1 & 0 & -\frac{3}{5} & 1/5 \\ 0 & 1 & -4/5 & 8/5 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\rightarrow x - \frac{3}{5}z = \frac{1}{5}$

$x = \frac{3}{5}z + \frac{1}{5}$

$y - \frac{4}{5}z = \frac{8}{5}$

$y = \frac{4}{5}z + \frac{8}{5}$

Let $z = t$ where t is any real number.
The parametric solution is $\left(\frac{3}{5}t + \frac{1}{5}, \frac{4}{5}t + \frac{8}{5}, t\right)$.
There are infinitely many solutions.

21. Turn to page 9.

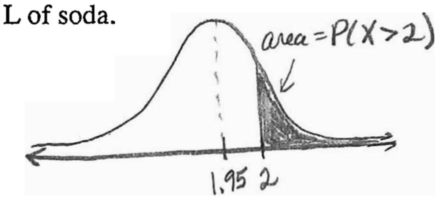
22. The true volume of soda in 2-liter bottles packaged by Acme, Inc. is normally distributed with a mean of 1.95 L and a standard deviation of 0.35 L.

(a) Find the probability that a randomly selected bottle contains more than 2 L of soda.

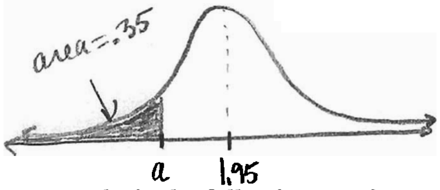
X= number of Liters of soda in the bottle.

$P(X>2) =$ normalcdf(2, 1E99, 1.95, 0.35)

$= \boxed{.4432}$



(b) 35% of all bottles contain a volume that is less than 1.8151 liters.



area μ σ

a= invnorm(.35, 1.95, 0.35)

$\boxed{a = 1.8151}$

23. Solve for x, y, and z in the following matrix equation:

$$\begin{bmatrix} 4 & x & 2z \\ 0 & 1 & 3 \end{bmatrix} \begin{bmatrix} 2 & y \\ -1 & 0 \\ 3 & 4 \end{bmatrix} + \begin{bmatrix} -8 & y \\ 0 & -2 \end{bmatrix}^T = \begin{bmatrix} -4 & 12 \\ 5 & 10 \end{bmatrix}$$

2×3 = 3×2

2×2

$$\begin{bmatrix} 4(2)+x(-1)+2z(3) & 4(y)+x(0)+2z(4) \\ 0(2)+1(-1)+3(3) & 0(y)+1(0)+3(4) \end{bmatrix} + \begin{bmatrix} -8 & 0 \\ y & -2 \end{bmatrix} = \begin{bmatrix} -4 & 12 \\ 5 & 10 \end{bmatrix}$$

$$\begin{bmatrix} 8-x+6z & 4y+8z \\ 8 & 12 \end{bmatrix} + \begin{bmatrix} -8 & 0 \\ y & -2 \end{bmatrix} = \begin{bmatrix} -4 & 12 \\ 5 & 10 \end{bmatrix}$$

$$\begin{bmatrix} -x+6z & 4y+8z \\ 8+y & 10 \end{bmatrix} = \begin{bmatrix} -4 & 12 \\ 5 & 10 \end{bmatrix}$$

$8 + y = 5$

$\boxed{y = -3}$

$4y + 8z = 12$

$4(-3) + 8z = 12$

$8z = 24$

$\boxed{z = 3}$

$-x + 6z = -4$

$-x + 6(3) = -4$

$-x = -22$

$\boxed{x = 22}$

24. How many ways can 4 red crayons, 6 blue crayons, and 5 yellow crayons be lined up in a row if crayons of the same color are identical?

$$\frac{15!}{4! \cdot 6! \cdot 5!} = \boxed{630,630}$$

25. Elliot wants to buy Olivia 8 movies for her birthday. Olivia has given Elliot a list of 15 different movies that she would like to have, but Elliot left the list at home. How many ways can Elliot randomly choose 8 movies off a shelf that has 35 different movies (15 of which are on Olivia's list) and get at least 7 movies that are on the list?

E - event that at least 7 movies in the sample are on the list.

$$n(E) = \underbrace{C(15,7)C(20,1)}_{\text{(Exactly 7)}} + \underbrace{C(15,8)}_{\text{(Exactly 8)}} = \boxed{135,135}$$

26. A random variable X has a mean of 34 and a standard deviation of 2.5. Use Chebychev's inequality to estimate $P(28.75 \le X \le 39.25)$. difference $= \sigma k$

$5.25 = 2.5k$

$2.1 = k$

$34 - 28.75 = 5.25$ $\quad$ $39.25 - 34 = 5.25$

$$P(28.75 \le X \le 39.25) \ge 1 - \frac{1}{(2.1)^2} = \boxed{\frac{341}{441}}$$

27. Let $U = \{a,b,c,d,e,1,2,3,4,5\}$, and let $A = \{a,c,e,3,5\}$, $B = \{1,2,4,d\}$, and $C = \{c,e,3\}$. Which of the following are true?

TRUE $\boxed{\text{(a) } A \text{ and } B \text{ are disjoint.}}$ $A \cap B = \emptyset$

TRUE $\boxed{\text{(b) } \{a,2,5\} \subseteq B \cup C^c}$ $C^c = \{a,b,d,1,2,4,5\}$ $\quad B \cup C^c = \{1,2,4,d,a,b,5\}$

FALSE (c) $e \subset C$ (Symbol should be $e \in C$)

(d) $\{c,e,3\} \in \{a,c,e,3,5\}$ FALSE (symbol should be $\subset$ or $\subseteq$)

$\boxed{\text{(e) } A \text{ has 32 subsets.}}$ $2^n = 2^5 = 32$ TRUE

(f) $(A \cap C)^c = \{3,5\}$ FALSE $\quad A \cap C = \{c,e,3\}$ $\quad (A \cap C)^c = \{a,b,d,1,2,4,5\}$

28. How much should Bob invest now in a savings account paying 2.25%/year compounded daily so that at the end of 10 years he has $25,000 in the account?

N = 365*10 $\quad$ PMT = 0

I% = 2.25 $\quad$ FV = 25000

PV = ? $\quad$ P/Y = C/Y = 365

$\boxed{\$19,963.04}$

29. According to company records, 24% of all customers at Acme Hardware on any particular day buy caulk. Find the probability that among 35 randomly selected customers, at most 7 buy caulk that day.

X = number of customers who buy caulk.

ANSWER:

$\boxed{P(X \leq 7) = .3728}$

Find $P(X \leq 7)$.

Binomial
1) $n = 35$
2) two outcomes; "success" = buy
3) $p = .24$
4) Indep? yes.

WORK:

BINOM Program	binomcdf(35, .24, 7)
$n = 35$ $p = .24$ Option 2 LowerK = 0, upperK = 7	

30. (Section 9.4) Roy and Clarice play a game in which they each flip a coin at the same time. If both coins land heads, Roy pays Clarice \$2. If both coins land tails, then Clarice pays Roy \$3. If Roy gets heads and Clarice gets tails, Clarice pays Roy \$5. Otherwise, Roy pays Clarice \$6.

(a) Write the payoff matrix for this two-person zero-sum game.

Clarice

$$\text{Roy} \quad \begin{array}{c c} & \begin{array}{cc} H & T \end{array} \\ \begin{array}{c} H \\ T \end{array} & \begin{bmatrix} \underline{-2} & 5 \\ \underline{-6} & 3 \end{bmatrix} \end{array}$$

(b) Find the maximin and minimax strategies for the row and column players, respectively.

maximin strategy for Roy: Row 1
minimax strategy for Clarice: Column 1

(c) Is this game strictly determined? If yes, find the saddle point and state the value of the game.

Since there is a saddle point at $a_{11} = -2$, this game is strictly determined. The value of the game is -2 and favors Clarice.