Week in Review # 2
Sections 1.2, 1.3, and 2.1

Things to know:
- How to work with lines: interpreting slope, finding linear equations
- How to perform linear regression on your calculator
- How to work with functions: definition, evaluation, graphical information, finding domain and range
- How to find cost, price, revenue, and profit functions

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1. Find an equation for the following lines:

   (a) The line passing through $(-2, 3)$ with a y-intercept of 1. $(0,1)$

   $m = \frac{1-3}{0-(-2)} = \frac{-2}{2} = -1$

   $\boxed{y = -x + 1}$

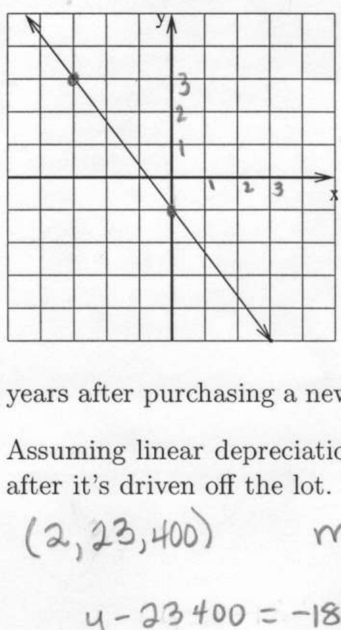
   (b) The line with x-intercept -4 and slope $\frac{5}{2}$.

   $(-4, 0)$, $m = \frac{5}{2}$

   $y - 0 = \frac{5}{2}(x + 4)$

   $\boxed{y = \frac{5}{2}x + 10}$

   (c)



   $(0, -1)$ y-int.

   $m = \frac{\Delta y}{\Delta x} = \frac{4}{-3}$

   $\boxed{y = -\frac{4}{3}x - 1}$

2. Two years after purchasing a new car, it is worth \$23,400. Three years later, it is worth \$5400 less.

   (a) Assuming linear depreciation, find a linear function that models the value of the car x years after it's driven off the lot.

   x = years
   y = value of car

   $(2, 23{,}400)$ $m = \frac{\Delta y}{\Delta x} = \frac{-5400}{3} = -1800$

   $y - 23400 = -1800(x - 2)$

   $y = -1800x + 3600 + 23400$

   $y = -1800x + 27000$ dollars, where x is the number of yrs since it was purchased.

   (b) How much was the car worth brand new? $x = 0$

   $y = -1800(0) + 27000$

   $\boxed{y = 27000 \text{ dollars}}$

   (c) What is the life expectancy of the car? (i.e. When is it expected to be worth nothing?)

   Set $y = 0$

   $0 = -1800x + 27000$

   $1800x = 27000$

   $\boxed{x = 15 \text{ years from purchase}}$

3. Let $C_1(x) = 0.35x + 2450$ be the cost of producing x doodads and $C_2(x) = 0.20x + 3500$ the cost of producing x trinkets.

(a) How much does it cost to produce 3000 doodads? 3000 trinkets?

$C_1(3000) = 0.35(3000) + 2450 = \boxed{\$3500}$

$C_2(3000) = 0.20(3000) + 3500 = \boxed{\$4100}$

(b) Which costs more *per item* to produce?

Doodads– larger variable cost (\$0.35 per doodad vs. \$0.20 per trinket)

(c) Which item costs more to start up?

trinkets - \$3500 fixed costs vs. only \$2450 in fixed costs for doodads.

4. Bevo BBQ, Co. determined that the variable cost of producing their BBQ sauce is 50 cents per bottle and the fixed costs are \$250. They charge \$5 per bottle of sauce.

(a) What is the company's cost function for their BBQ sauce?

$C(x) = 0.5x + 250$ dollars, where x is the # of bottles produced.

(b) Their revenue function?

$R(x) = 5x$ dollars, where x is # of bottles sold.

(c) Their profit function?

$P(x) = R(x) - C(x) = 5x - (0.5x + 250)$

$= 5x - 0.5x - 250$

$P(x) = 4.5x - 250$ dollars, where x is # of bottles produced + sold

(d) If they produce 5000 bottles of sauce, what is the company's cost? Revenue? Profit?

$C(5000) = 0.5(5000) + 250 = 2500 + 250 = \boxed{\$2750}$

$R(5000) = 5(5000) = \boxed{\$25000}$

$P(5000) = R(5000) - C(5000)$

$= 25000 - 2750 = \boxed{\$22,250}$

5. The table below gives data for total sales (in thousands of dollars) of *Newspaper XYZ* through the first 10 weeks of the year.

Week	1	2	3	4	6	8	10
Total sales (K)	33	62	98	121	174	235	307

Find a linear regression model for this data and use it to answer the following:

(a) During what week will total sales hit one million dollars?

Model: $y = 29.6982x + 2.8943$ thousand dollars, where x is the # of weeks since beginning of yr.

$\$1{,}000{,}000 = \$1{,}000$ thousand so $y = 1000$

$1000 = 29.6982x + 2.8943$

$997.1057 = 29.6982x$

$x = 33.5746$

Sometime during 34th wk

(b) What are their weekly sales?

money earned per week

rate of change: SLOPE!

$m = 29.6982$ so about $29,698.20 in sales per week.

6. The table below provides data on the prices at which x items will be either demanded or supplied.

x	50	100	150	200	250
price – demand($)	27	25	23	20	17.5
price – supply($)	1.5	2.5	3.5	5	6.25

Find linear regression models for this data and use them to predict the equilibrium point.

where demand = supply

price-demand

Store in Y_1: $p = -0.048x + 29.7$ dollars per unit, where x is the number of items demanded.

Price-supply

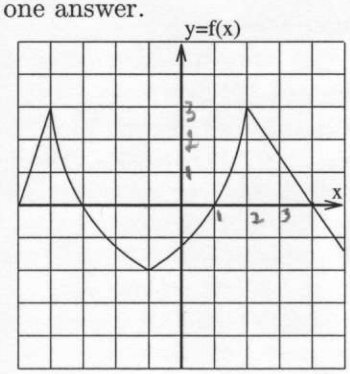
Store in Y_2: $p = 0.024x + 0.15$ dollars per unit, where x is the number of units supplied.

Use calculator to find intersection point.

Equil. Point $= (410.41667, 10)$

Equilibrium quantity ≈ 410, Equilibrium price = $10.

7. Use the following graph of a function f to estimate x or y. Some problems may have more than one answer.



(a) $y = f(-1) = \boxed{-2}$

(b) $3 = f(x)$ $\boxed{x=2,\ x=-4}$

(c) $f(x) \geq 0$ $[-5,-3] \cup [1,4]$ ← these are x values for which $f(x) \geq 0$

8. Find the domain of the following functions:

(a) $f(x) = \sqrt[6]{5 - 11x}$

Even Indexed root

$$5 - 11x \geq 0$$
$$-11x \geq -5$$
$$x \leq \frac{5}{11}$$

$\boxed{\left(-\infty, \frac{5}{11}\right]}$

(b) $g(x) = \dfrac{\sqrt[3]{x - 3}}{x^3 - 8x^2 + 15x}$ can't be 0

odd (all $\mathbb{R}$)

$$x^3 - 8x^2 + 15x = 0$$
$$x(x^2 - 8x + 15) = 0$$
$$x(x-5)(x-3) = 0$$

$x = 0$ $\quad x - 5 = 0$ $\quad x - 3 = 0$

$x = 5$ $\quad x = 3$

Domain: $\boxed{(-\infty, 0) \cup (0,3) \cup (3,5) \cup (5, \infty)}$

(c) $h(x) = \dfrac{\sqrt{x - 3}}{x^3 - 8x^2 + 15x}$

Same denominator, so

x can't be 0, 3, 5, but $x - 3 \geq 0$

Not ≥ 3

$x \geq 3$

Not inclusive on 3 since $x = 3$ makes denom. $= 0$

$(3,5) \cup (5, \infty)$

9. For the following functions, find:

   i. $f(a+h)$

   ii. $f(a+h) - f(a)$

   iii. $\dfrac{f(a+h) - f(a)}{h}$

   (a) $f(x) = \frac{1}{x^2}$

i) $f(a+h) = \dfrac{1}{(a+h)^2}$

ii) $f(a+h) - f(a) = \dfrac{1}{(a+h)^2} - \dfrac{1}{a^2}$

$= \dfrac{a^2}{a^2(a+h)^2} - \dfrac{(a+h)^2}{a^2(a+h)^2}$

$= \dfrac{a^2 - (a^2+2ah+h^2)}{a^2(a+h)^2}$

$= \dfrac{\cancel{a^2} - \cancel{a^2} - 2ah - h^2}{a^2(a+h)^2}$

$= \dfrac{-2ah-h^2}{a^2(a+h)^2}$

iii) $\dfrac{f(a+h)-f(a)}{h} = \dfrac{\left(\dfrac{-2ah-h^2}{a^2(a+h)^2}\right)}{h}$

$= \left(\dfrac{-2ah-h^2}{a^2(a+h)^2}\right)\dfrac{1}{h}$

$= \dfrac{\cancel{h}(-2a-h)}{\cancel{h}a^2(a+h)^2} = \boxed{\dfrac{-2a-h}{a^2(a+h)^2}}$

(b) $f(x) = \sqrt{3-x}$

i) $f(a+h) = \sqrt{3-(a+h)} = \sqrt{3-a-h}$

ii) $f(a+h) - f(a) = \sqrt{3-a-h} - \sqrt{3-a}$

iii) $$\frac{f(a+h)-f(a)}{h} = \left(\frac{\sqrt{3-a-h} - \sqrt{3-a}}{h}\right)\underbrace{\left(\frac{\sqrt{3-a-h} + \sqrt{3-a}}{\sqrt{3-a-h} + \sqrt{3-a}}\right)}_{\text{This is a form of the number 1.}}$$

$$= \frac{(3-a-h) - (3-a)}{h\left(\sqrt{3-a-h} - \sqrt{3-a}\right)}$$

$$= \frac{\cancel{3} - \cancel{a} - \cancel{h} - \cancel{3} + \cancel{a}}{\cancel{h}\left(\sqrt{3-a-h} - \sqrt{3-a}\right)}$$

$$= \boxed{\frac{-1}{\sqrt{3-a-h} - \sqrt{3-a}}}$$