Math 141 - Week in Review #1

Section 1.2

- $m = \dfrac{\Delta y}{\Delta x} = \dfrac{y_2 - y_1}{x_2 - x_1}$
- Point-Slope Form: $y - y_1 = m(x - x_1)$, where m is slope and (x_1, y_1) is *any* point on the line
- Slope-Intercept Form: $y = mx + b$
- General Form: $Ax + By + C = 0$
- Parallel Lines - Two distinct lines are parallel if and only if ther slopes are equal or their slopes are both undefined.
- Perpendicular Lines - slopes are opposite (or negative) reciprocals of each other
- Equation of a Vertical Line: $x = a$
- Equation of a Horizontal Line: $y = b$

Section 1.3

- Total Cost Function: $C(x) = cx + F$ where c = production cost per unit, F = fixed cost, and x = number of units produced
- Revenue Function: $R(x) = sx$ where s = selling price per unit and x = number of units sold
- Profit Function: $P(x) = R(x) - C(x)$
- For Demand and Supply Functions: x = quantity demanded or supplied and y = unit price
- Demand equations have negative slope.
- Supply equations have positive slope.

Section 1.4

- Break-even Point: the point (x_0, y_0) where *revenue equals cost*, i.e., $R(x) = C(x)$

  x_0 = break-even quantity

  y_0 = break-even revenue
- Market Equilibrium: occurs when the quantity demanded equals the quantity supplied (i.e., demand = supply)

Section 1.5

- Equation for the least-squares regression line - computed in your calculator using the command LinReg(ax+b) L_1, L_2, Y_1

1. Find the equation of the line that passes through the point $\overset{(x_1,y_1)}{(-2,5)}$ and

   (a) the point $\overset{(x_2,y_2)}{(4,-3)}$.

$(-2,5),(4,-3)$ $\quad m = \dfrac{-3-5}{4+2} = \dfrac{-8}{6} = -\dfrac{4}{3}$

point-slope form: $y - y_1 = m(x - x_1)$

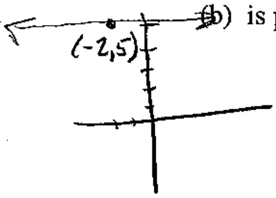
$y - 5 = -\frac{4}{3}(x+2)$

$y = -\frac{4}{3}x - \frac{8}{3} + 5\cdot\frac{3}{3}$

$15 + 8 = 23$

slope-intercept form: $\boxed{y = -\frac{4}{3}x + \frac{7}{3}}$

   (b) is parallel to the x-axis.



horizontal line

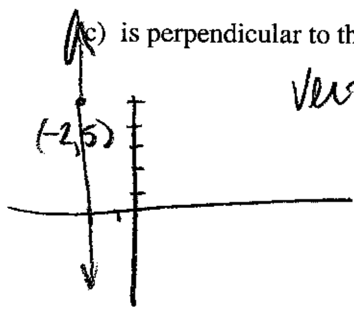
$y = b$

↑

y-intercept

$\boxed{y = 5}$

   (c) is perpendicular to the x-axis.



vertical line: $x = a \leftarrow$ x-intercept

$\boxed{x = -2}$

2. Let L_1 be the line that passes through the points $(7,z)$ and $(-3,-1)$ and let L_2 be the line that passes through the points $(5,-4)$ and $(-2,6)$. Find the value of z so that

   (a) L_1 is parallel to L_2.

slopes are equal

$m_1 = \dfrac{-1-z}{-3-7} = \dfrac{-1-z}{-10}$

$m_2 = \dfrac{6-(-4)}{-2-5} = \dfrac{10}{-7}$

set equal

$\dfrac{-1-z}{-10} = \dfrac{10}{-7}$ $\rightarrow$ $-7(-1-z) = -100$

$7 + 7z = -100$

$7z = -107$

$\boxed{z = -\frac{107}{7}}$

   (b) L_1 is perpendicular to L_2.

slopes are opposite reciprocals

$m_1 = -\frac{1}{m_2}$ ← flip m_2 + change the sign.

$\dfrac{-1-z}{-10} = \dfrac{7}{10}$ $\longrightarrow$ $10(-1-z) = -70$

$-10 - 10z = -70$

$-10z = -60$

$\boxed{z = 6}$

3. A local pool had 150 visitors on a day when the outside temperature reached a high of 98°F, but when the high dropped to 83°F, only 96 visitors came.

$y=f(x)$ means y is a function of x

(a) Assuming a linear relationship, find an equation that gives the number of visitors as a function of the high temperature for the day.

y x

$(x, y) = (\text{temp}, \# \text{ of visitors})$

$(98, 150)$

$(83, 96)$

$m = \dfrac{96-150}{83-98}$

$m = \dfrac{-54}{-15}$

$m = \dfrac{18}{5}$

$y - y_1 = m(x - x_1)$

$y - 150 = \frac{18}{5}(x - 98)$

$y = \frac{18}{5}x - 98\left(\frac{18}{5}\right) + 150$

$\boxed{y = \frac{18}{5}x - \frac{1014}{5}}$

(b) According to your answer in part (a), if the high temperature drops by 7°F, what happens to the number of visitors at the pool that day?

change in temperature

$\Delta x = -7$

$y = 3.6x - 202.8$

$m = \dfrac{\Delta y}{\Delta x} = \dfrac{\text{change in \# of visitors}}{\text{change in temp}}$

$\dfrac{\Delta y}{-7} = \dfrac{18}{5} \longrightarrow 5\Delta y = (-7)(18)$

$5\Delta y = -126$

$\Delta y = -25.2$

There will be 25 fewer visitors to the pool

(c) According to your answer in part (a), if 15 more people visit the pool on one day than on the previous day, how does the temperature on that day compare to the temperature the day before?

Δy

$m = \dfrac{\Delta y}{\Delta x} = \dfrac{15}{\Delta x} = \dfrac{18}{5}$

$18\Delta x = 75$

$\Delta x = 75/18 = 4.1667$

The temperature is about 4.1667° higher than on the day before

(d) For what high temperature can you expect no visitors at the pool?

find x

$y=0$

Equation:

$y = 3.6x - 202.8$

Solve for x:

$0 = 3.6x - 202.8$

$202.8 = 3.6x$

$56.3333 = x$

$\boxed{56.3333°F}$

(e) Does this mathematical model make sense for any value of x?

No. Since x represents the high temperature, any value much greater than the world record high would be unrealistic - humans can only survive for so long in certain temperatures. Also, water boils at 212°F, so definitely nothing higher than that is possible. No temp. lower than 56.3333° F would make sense either since that would result in a neg. # of people at the pool.

4. A car is purchased for \$35,000, and its value depreciates to \$21,500 in just three years.

(a) If we assume that the car depreciates in value at a constant rate each year, find the rate of depreciation.

Def: Slope is a measure of the rate of change of y (value) with respect to x (time)

(time, value)

(0, 35000)

(3, 21500)

$$m = \frac{21500-35000}{3-0}$$

$$m = \frac{-13500}{3}$$

$$m = -4500$$

$\boxed{\text{The car depreciates in value by \$4500 each year.}}$

(b) Find a function that models the value of the car at time t.

$$y = mt+b$$

$$\boxed{y = -4500t + 35000}$$

(c) According to your model, when will the car be worth \$10,000?

Find t when $y = 10000$

$$10000 = -4500t + 35000$$

$$-25000 = -4500t$$

$$\frac{-25000}{-4500} = t$$

$$5.5556 \approx \frac{50}{9} = t$$

$\boxed{\text{In about 5.5556 years}}$

(d) What is the domain of the function found in part (b)?

domain – the set of x-values (or in this example, t-values) that you are allowed to plug in to the function.

minimum t: $t=0$

Maximum t: when $y=0$

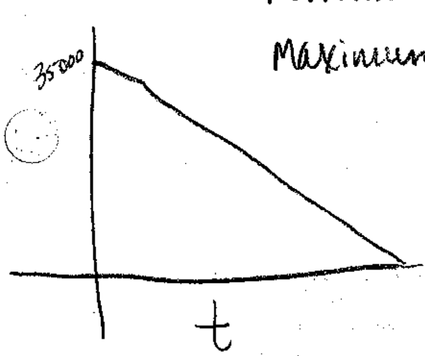
$$0 = -4500t + 35000$$

$$-35000 = -4500t$$

$$t = \frac{-35000}{-4500} = \frac{70}{9} \approx 7.7778$$

$\boxed{\text{Domain: } 0 \le t \le \frac{70}{9}}$

(values of t between 0 and $\frac{70}{9}$, incl.)



5. RB, Inc. incurs a production cost of $20 for each pair of rollerblades it makes and then sells each pair for $75. If the company has a total cost of $5,800 when no pairs of rollerblades are made, what is RB, Inc.'s break-even point?

total cost

$C(x) = cx + F$

$C(x) = 20x + F$

$5800 = 20(0) + F$

$5800 = F$

$C(x) = 20x + 5800$

$R(x) = sx$

$R(x) = 75x$

$20x + 5800 = 75x$

$5800 = 55x$

$105.4545 \approx \frac{5800}{55} = x$

$R(x) \overset{\text{set}}{=} C(x)$

$R\left(\frac{5800}{55}\right) = 75\left(\frac{5800}{55}\right)$

$R\left(\frac{5800}{55}\right) = \frac{87000}{11}$

≈ 7909.09

$\boxed{\text{Break-even point: } (105.4545,\ 7909.09)}$

Check w/ instructor about rounding.

6. Acme, Inc. loses $3,000 per month when 75 gizmos are produced and sold per month.

(a) If each gizmo costs $50 to produce and then sells for $80, find the total cost, revenue, and profit functions for Acme, Inc.'s gizmo sales.

$C(x) = cx + F$

$R(x) = sx$

$P(x) = R(x) - C(x)$

$c = 50$

$s = 80$

When $x = 75$, profit $= -3000$

$C(x) = 50x + F$

$R(x) = 80x$

$P(x) = 80x - (50x + F)$

$P(x) = 80x - 50x - F$

$P(x) = 30x - F$

$-3000 = 30(75) - F$

$5{,}250 = F$

$\boxed{C(x) = 50x + 5{,}250}$

$\boxed{R(x) = 80x}$

$\boxed{P(x) = 30x - 5{,}250}$

(b) What is the profit (or loss) associated with producing and selling 150 gizmos?

$x = 150$ find $P(150)$

$P(150) = 30(150) - 5250$

$= 4500 - 5250$

$P(150) = -750$

$\boxed{\text{loss of \$750}}$

(Profit $= -\$750$)

(c) What does the slope of the total cost function represent?

$C(x) = \underset{\uparrow\ \text{slope}}{50}x + 5250$

$m = \frac{\Delta y}{\Delta x} = 50 = \frac{50}{1}$

When $\Delta x = 1$, $\Delta y = 50$.

When one additional unit is produced, total cost increases by \$50.

$m = \$50$ represents the marginal cost – the cost of producing one additional unit.

7. When a movie theater charges \$5 per ticket, an average of 750 tickets are sold per day during the week. When the theater increases each ticket price by \$2, 150 fewer tickets are sold.

(a) Find the demand equation, assuming it is linear.

$(x, y) = $ (quantity demanded, unit price)

$(750, 5)$

$m = \frac{\Delta y}{\Delta x} = \frac{\text{change in price}}{\text{change in \#demanded}} = \frac{2}{-150} = -\frac{1}{75}$

$y - y_1 = m(x - x_1)$

$y - 5 = -\frac{1}{75}(x - 750)$

$y = -\frac{1}{75}x + 10 + 5$

$\boxed{p = -\frac{1}{75}x + 15}$

(b) How many people would attend each weekday if tickets were free?

unit price = 0

$0 = -\frac{1}{75}x + 15$

$-15 = -\frac{1}{75}x$

$(75)(15) = x$

$\boxed{x = 1125 \text{ people}}$

(c) If the supply equation for movie tickets is $x - 98p + 294 = 0$, find the equilibrium quantity and equilibrium price for the movie tickets.

Demand = supply

$-98p = -x - 294$

$p = \frac{1}{98}x + 3$

$\frac{1}{98}x + 3 = -\frac{1}{75}x + 15$

$\frac{173}{7350}x = 12$

$x = \frac{88200}{173} \approx 509.8266$

$p = \frac{1}{98}\left(\frac{88200}{173}\right) + 3$

$p = \frac{1419}{173} \approx \$8.20$

Equilib. quant ≈ 510 tickets
Equil. price ≈ \$8.20

8. When the unit price for a particular digital camera is \$100, consumers purchase 10,000 units. If the unit price increases by \$50, the number of cameras demanded decreases by 2,500. Manufacturers will not supply any of these cameras when the unit price is \$65, but when the unit price is \$200, manufacturers will supply 25,000 cameras. How many cameras are produced at market equilibrium?

Supply = Demand

Demand

$(10000, 100)$

$m = \frac{\Delta y}{\Delta x} = \frac{50}{-2500} = -\frac{1}{50}$

$y - 100 = -\frac{1}{50}(x - 10000)$

$y = -\frac{1}{50}x + 200 + 100$

$y = -\frac{1}{50}x + 300$

Supply

$(0, 65)$

$(25000, 200)$

$m = \frac{200-65}{25000-0} = \frac{135}{25000} = \frac{27}{5000}$

$y = \frac{27}{5000}x + 65$

$-\frac{1}{50}x + 300 = \frac{27}{5000}x + 65$

$235 = \frac{127}{5000}x$

$\frac{1{,}175{,}000}{127} = x$

$x \approx 9251.9685$

Approx. $\boxed{9252 \text{ cameras}}$

9. Many of the businesses in a small town participated in a fundraiser for the American Cancer Society. Each business formed a fundraising team from its own employees. The following table gives the number of employees on each team and the corresponding amount of money raised (in thousands of dollars).

Number of Team Members (x)	15	19	21	22	26	30	30	31
Amount of Money Raised (y)	0.95	1.1	1.9	2.5	3.5	4.7	5.1	6

(a) Determine the equation of the least-squares regression line for these data. Round to four decimal places.

$y = 0.3146x - 4.4110$

(b) Does the regression line give a good representation of the data? Why or why not?

Yes- the correlation coefficient $r = .9734$, which is very close to 1.

or- look at scatter diagram + regression line - data appear to be linear.

xmin 10
xmax 35
xscl 2
ymin 0
ymax 7
yscl 1

(c) How many members would you expect a team which raised $3,200 to have?

find x

Amt of \$ $\rightarrow y = 3.2$

$3.2 = 0.3146x - 4.4110$

$7.6110 = 0.3146x$

$24.1926 \approx x$

$\boxed{\text{about 24 members}}$

$y_2 = 3.2$
Intersect
$x = 24.1904\ldots$

(d) How much money would you expect a team with 18 members to be able to raise?

$x = 18$ find y

$y = 0.3146(18) - 4.4110$

$y = 1.2581 \longrightarrow \boxed{\$1,258.10}$

or 2^{nd} TRACE (calc)
option 1: value
$X = 18$
$y = 1.2523 \rightarrow \boxed{\$1252.30}$

slightly different

(e) If a business adds one additional member to its team, in what way can it expect the amount of money the team will raise to change? Be specific.

$\Delta X = 1$
find Δy

$m = \frac{\Delta y}{\Delta x} = 0.3146$

$\frac{\Delta y}{1} = 0.3146$

$\Delta y = 0.3146$

$\boxed{\text{They can expect to raise \$314.60 more}}$