Math 166 - Week in Review #2

Section 2.3 - Systems of Linear Equations: Underdetermined and Overdetermined Systems

- Using RREF to Solve Systems of Equations
  STEP 1: Check the final matrix to see if there is no solution. (If the system has no solution, state so and stop here. Otherwise, go on to Step 2.)
  STEP 2: Circle the leading 1's.
  a) If each variable has a leading 1 in its column, then there is a unique solution.
  b) Otherwise, there are (potentially) multiple solutions and each variable not having a leading one in its column is a parameter. **NOTE: If the variables in the system of equations represent quantities or units of some items, then you must consider whether it is necessary to put restrictions on the parameters (and do so if it is necessary).**

- Overdetermined systems have more equations than unknowns. These systems can have a unique solution, infinitely many solutions, or no solution.

- Underdetermined systems have fewer equations than unknowns. These systems can only have infinitely many solutions or no solution.

- If a system of equations has infinitely many solutions, you should represent the solutions in parametric form.

Section 2.4 - Matrices

- The size of a matrix is always *number of rows* $\times$ *number of columns*.

- c_{ij} represents the entry of the matrix C in row i and column j.

- To add and subtract matrices, they must be the same size.

- When adding or subtracting matrices, add or subtract corresponding entries.

- A scalar product is computed by multiplying each entry of a matrix by a scalar (a number).

- Transpose - The rows of the matrix A become the columns of A^T.

Section 2.5 - Multiplication of Matrices

- The matrix product AB can be computed only if the number of columns of A equals the number of rows of B.

- If $C = AB$, then c_{ij} is computed by multiplying the i^{th} row of A by the j^{th} column of B.

- Identity Matrix - Denoted by I_n, the identity matrix is the $n \times n$ matrix with 1's down the main diagonal (from upper left corner to lower right corner) and 0's for all other entries.

- If A is $m \times n$, then $AI_n = A$ and $I_mA = A$.

1. Solve the following systems of equations. If there are infinitely many solutions, state so and give the parametric solution. If there is no solution, state so.

   (a) $$\begin{aligned} 3x + 4y - z &= -8 \\ 2x + 5y + z &= -3 \end{aligned}$$

   (b) $$\begin{aligned} x + 2y &= 3 \\ 3x + 3y &= 7 \\ 2x + y &= 4 \end{aligned}$$

2. For the next two word problems do the following:
   I) Define the variables that are used in setting up the system of equations.
   II) Set up the system of equations that represents this problem.
   III) Solve for the solution.
   IV) If the solution is parametric, then tell what restrictions should be placed on the parameter(s). Also give three specic solutions.

   (a) The management of a private investment club has a fund of $300,000 earmarked for investment in stocks. To arrive at an acceptable overall level of risk, the stocks that the management is considering have been classified into three categories: high-risk, medium-risk, and low-risk. Management estimates that high-risk stocks will have a rate of return of 16 percent per year; medium-risk stocks, 10 percent per year; and low-risk stocks, 4 percent per year. The investment in medium-risk stocks is to be twice the investment in stocks of the other two categories combined. If the investment goal is to have an average rate of return of 11 percent per year on the total investment, determine how much the club should invest in each type of stock.

(b) A convenience store sold 23 sodas one summer afternoon in 12-, 16-, and 20-ounce cups (small, medium, and large). The total volume of soda sold was 376 ounces, and the total revenue was \$48. If the prices for small, medium, and large sodas are \$1, \$2, and \$3 respectively, how many of each size did the store sell that day? (pg. 70-72, *Finite Mathematics* by Lial, et. al.)

3. Let $A = \begin{bmatrix} 4 & x \\ 3 & -7 \end{bmatrix}$, $B = \begin{bmatrix} 2 & -3 & 0 \\ 8 & k & -6 \end{bmatrix}$, $C = \begin{bmatrix} 4 & -5 \\ 0 & b \\ 7 & -10 \end{bmatrix}$, and $D = \begin{bmatrix} 5 & -1 & 3a \\ 2 & 6 & 4 \end{bmatrix}$. Compute each of the following:

(a) $B + 3D$

(b) $2C + B$

(c) $4D - 3C^T$

(d) $4a_{21} - 2c_{32} + 7d_{13}$

(e) DB

(f) $B^T DA$

(g) CD^T

(h) BB^T

(i) A^2

4. Solve for x and y:

$$3\begin{bmatrix} 2 & x \\ 5y & -1 \end{bmatrix} - \begin{bmatrix} -6 & 1 \\ 3y & -5 \end{bmatrix}^T = \begin{bmatrix} 12 & -7 \\ -2x & 2 \end{bmatrix}$$

5. The times (in minutes) required for assembling, testing, and packaging large and small capacity food processors are shown in the following table:

	Assembling	Testing	Packaging
Large	45	15	10
Small	30	10	5

(a) Define a matrix T that summarizes the above data.

(b) Let $M = \begin{bmatrix} 100 & 200 \end{bmatrix}$ represent the number of large and small food processors ordered, respectively. Find MT and explain the meaning of its entries.

(c) If assembling costs \$3 per minute, testing costs \$1 per minute, and packaging costs \$2 per minute, find a matrix C that, when multiplied with T, gives the total cost for making each size of food processor.