Math 142 - Week in Review #2

1. Find the domain of the following functions.

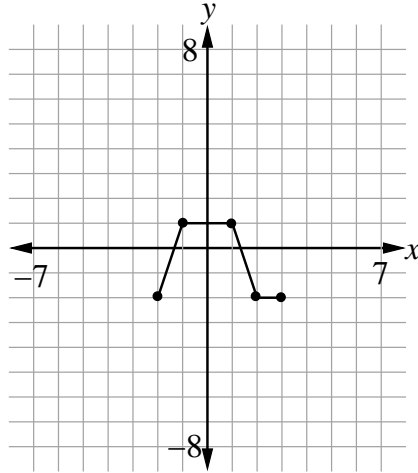   (a) $f(x) = \sqrt{x^2 - 9}$

   (b) $g(x) = \dfrac{1}{\sqrt[3]{x^2 + 3x - 28}}$
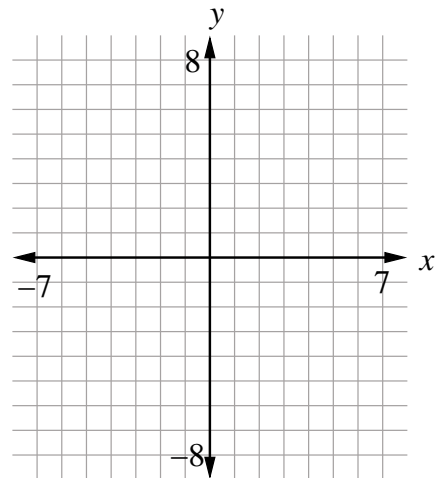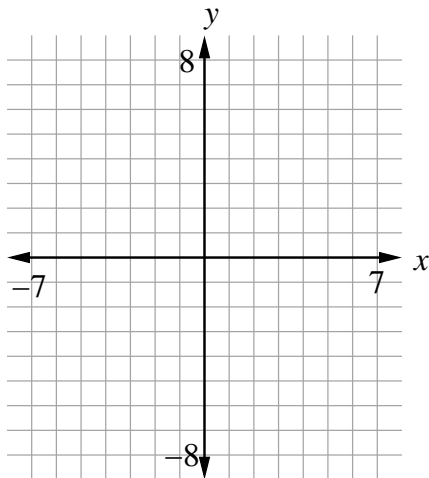
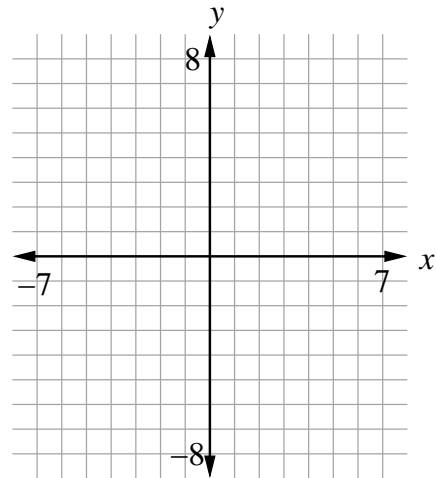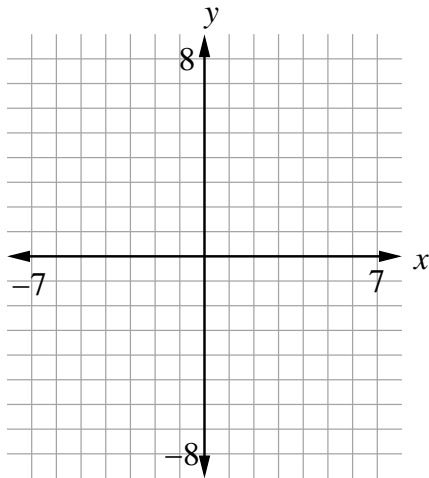   (c) $h(x) = \dfrac{1}{\sqrt{x^2 + 3x - 28}}$

2. If $f(x) = 3x^2 - 7$, find $\dfrac{f(4+h) - f(4)}{h}$.

3. Below is the graph of a function f. Use it to graph each of the following.

(a) $f(x-2)$

(b) $3 - f(x)$

(c) $2f(x)$

(d) $-f(x+1) - 2$

4. Indicate verbally how the graph of each of the following is related to the graph of one of the six basic functions shown in the text.

   (a) $g(x) = -2\,|\,x-5\,|+4$

   (b) $h(x) = 0.1\sqrt{x+3} - 8$

5. For each of the following, the graph of the function g is formed by applying the indicated sequence of transformations to the given function f. Find an equation for the function g.

   (a) The graph of $f(x) = \sqrt[3]{x}$ is shifted 3 units to the left, reflected about the x-axis, and then shifted 2 units up.
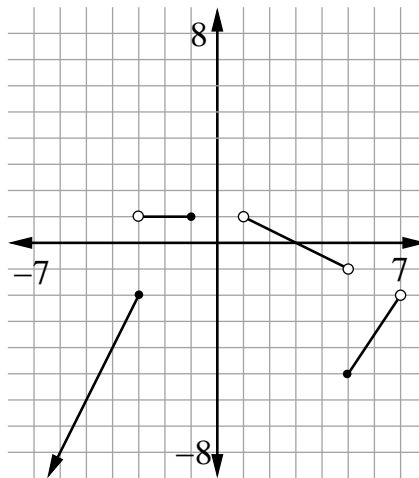
   (b) The graph of $f(x) = x^3$ is shifted 1 unit to the right, stretched vertically by a factor of 2, reflected about the x-axis, and shifted down 5 units.

6. Consider the function $k(x) = \begin{cases} |\,x+2\,|+3 & \text{if } x \le 1 \\ -x+4 & \text{if } x > 1 \end{cases}$.

   (a) Find $k(1)$ and $k(3)$.

   (b) Graph $k(x)$.

7. Use the function f, whose graph is shown below, to answer each of the following.

(a) Find $f(-4)$, $f(-3)$, $f(1)$, and $f(5)$.

(b) Find the values of x such that $f(x) = -2$.

(c) Find the values of x such that $f(x) < -2$.

(d) Find the values of x such that $f(x)$ is positive.

(e) Find the domain and range of $f(x)$.

(f) Write a piecewise-defined function for $f(x)$.

8. Find the domain of the function $f(x) = \begin{cases} \frac{3x+2}{x^2-4} & \text{if } x < 1 \\ \frac{x^2+x-12}{x-3} & \text{if } x \geq 2 \end{cases}$.

9. At an internet cafe, internet users pay a flat fee of $5 for access, and then additional charges are added based on the number of minutes of internet usage. The first 10 minutes are free, but for the next 30 minutes, a fee of $0.15 per minute is added. Any minutes used beyond this are charged at a rate of $0.10 per minute.

   (a) Write a piecewise-defined function that gives the amount owed by a person who uses the cafe's internet connection for x minutes.

   (b) How much is owed by a customer who used the internet for half an hour?

10. Find the x and y intercepts for each of the following.

    (a) $f(x) = 2x^2 + 2x - 24$

    (b) $g(x) = (x-3)^2 + 5$

11. For each of the following quadratic functions, find (i) the vertex, (ii) the vertex form of the equation of the quadratic function, (iii) the maximum or minimum value, and (iv) the range.

(a) $f(x) = -x^2 + 6x - 5$

(b) $g(x) = 2x^2 + 16x + 29$

12. Determine which of the following functions are polynomials. For those that are polynomials, describe their end behavior.

(a) $y = -\frac{1}{3}x^4 - 2x^{-2} + 5$

(b) $y = \pi x^5 + 3x^4 - 7x + \frac{1}{\pi}$

(c) $y = \sqrt{x} + 3x + 7$

(d) $y = -3x^2 + \frac{1}{e}x$

(e) $y = 7x^3 - 3x^2 + 2|x| + 7$

13. Acme Chairs, Inc., has determined the price-demand function for its recliner to be $p = 600 - 5x$ dollars per recliner. Acme has fixed costs that amount to $6,000 and variable costs of $130 per recliner.

(a) Find the price per recliner that maximizes revenue.

(b) What is Acme's maximum profit? How many recliners must be produced and sold to achieve the maximum profit?

(c) Find the break-even quantity (or quantities).

(d) For what production levels will Acme experience a loss?