Math 142 - Week in Review #2

1. Find the domain of the following functions.

   (a) $f(x) = \sqrt{x^2 - 9}$

$x^2 - 9 \geq 0$ Must make a signchart!

$(x-3)(x+3) \geq 0$

$x^2 - 9 = 0$ for $x = 3, -3$

	-5	-3	0	3	5
$(x-3)$	$-$		$-$		$+$
$(x+3)$	$-$		$+$		$+$
$(x-3)(x+3)$	$+$		$-$		$+$

← Plug test #'s into each piece and check sign

a) $\boxed{\text{Domain}: (-\infty, -3] \cup [3, \infty)}$

   (b) $g(x) = \dfrac{1}{\sqrt[3]{x^2 + 3x - 28}}$

Cuberoot - all real #'s, but in denominator

$x^2 + 3x - 28 = 0$

$(x+7)(x-4) = 0 \rightarrow x \neq -7, 4$

b) $\boxed{\text{Domain}: (-\infty, -7) \cup (-7, 4) \cup (4, \infty)}$

   (c) $h(x) = \dfrac{1}{\sqrt{x^2 + 3x - 28}}$

Even-indexed root, so $x^2 + 3x - 28 \geq 0$, but in denom., so can't equal 0:

$(x+7)(x-4) > 0$

	-10	-7	0	4	10
$(x+7)$	$-$		$+$		$+$
$(x-4)$	$-$		$-$		$+$
$(x+7)(x-4)$	$+$		$-$		$+$

c) $\boxed{\text{Domain}: (-\infty, -7) \cup (4, \infty)}$

2. If $f(x) = 3x^2 - 7$, find $\dfrac{f(4+h) - f(4)}{h}$.

$f(4+h) = 3(4+h)^2 - 7$

$= 3(16 + 8h + h^2) - 7$

$= 48 + 24h + 3h^2 - 7$

$= 41 + 24h + 3h^2$

$f(4) = 3(4)^2 - 7$

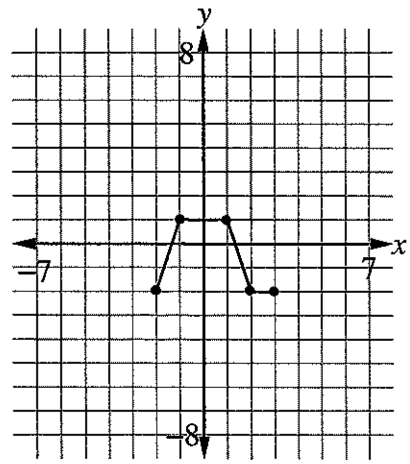
$f(4) = 41$

$f(4+h) - f(4) = 41 + 24h + 3h^2 - 41$

$= 24h + 3h^2$

$$\frac{f(4+h) - f(4)}{h} = \frac{24h + 3h^2}{h} = \frac{h(24 + 3h)}{h} = \boxed{24 + 3h}$$

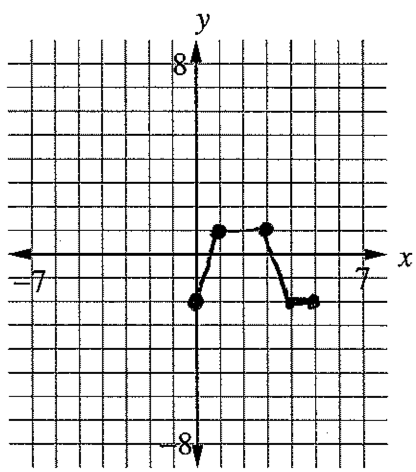
3. Below is the graph of a function f. Use it to graph each of the following.

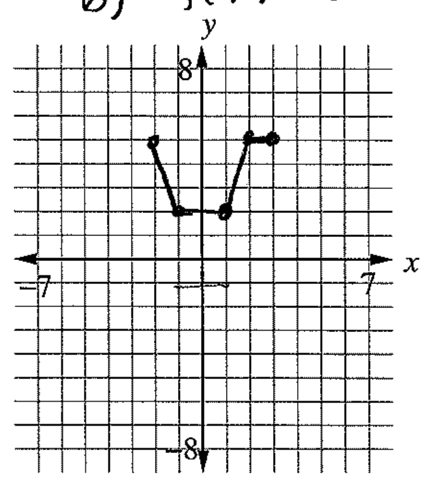


(a) $f(x-2)$ shift right 2 units
(b) $3-f(x)$
(c) $2f(x)$
(d) $-f(x+1)-2$

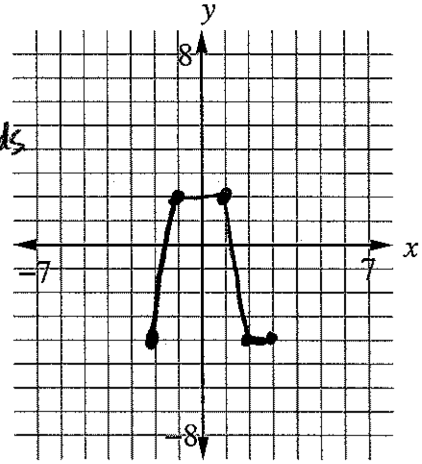
b) $-f(x)+3$ reflect about x-axis, then shift up 3

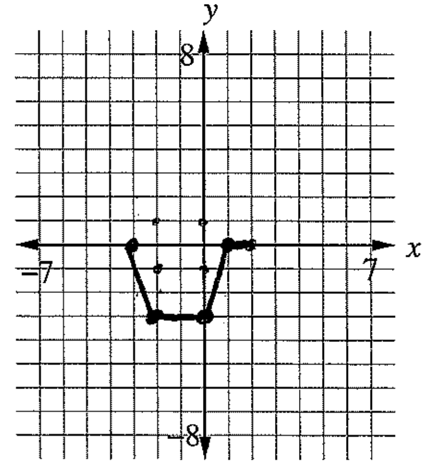
a)





c) $2f(x)$
multiply all y-coords by 2





d) $-f(x+1)-2$ shift left 1, reflect about x-axis, shift down 2.

4. Indicate verbally how the graph of each of the following is related to the graph of one of the six basic functions shown in the text.

   (a) $g(x) = -2|x-5|+4$ Shift $f(x) = |x|$ right 5 units, then stretch vertically by a factor of 2, the reflect about the x-axis, and then shift up 4 units.

   (b) $h(x) = 0.1\sqrt{x+3} - 8$

   Shift $f(x) = \sqrt{x}$ to the left 3 units, shrink vertically by multiplying each y-coord. by 0.1, and then shift down 8 units.

5. For each of the following, the graph of the function g is formed by applying the indicated sequence of transformations to the given function f. Find an equation for the function g.

   (a) The graph of $f(x) = \sqrt[3]{x}$ is shifted 3 units to the left, reflected about the x-axis, and then shifted 2 units up.

   $$g(x) = -\sqrt[3]{x+3} + 2$$

   (b) The graph of $f(x) = x^3$ is shifted 1 unit to the right, stretched vertically by a factor of 2, reflected about the x-axis, and shifted down 5 units.

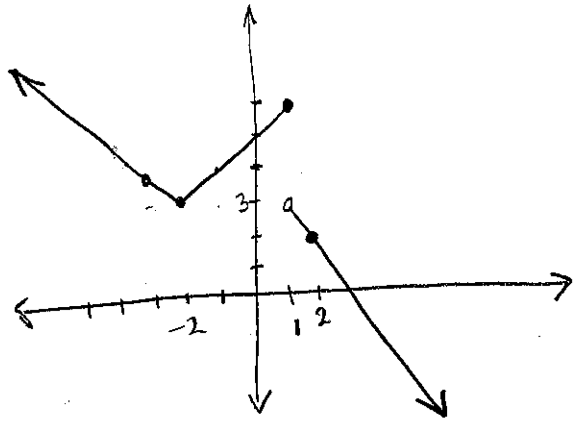
   $$g(x) = -2(x-1)^3 - 5$$

6. Consider the function $k(x) = \begin{cases} |x+2|+3 & \text{if } x \le 1 \\ -x+4 & \text{if } x > 1 \end{cases}$.

   (a) Find $k(1)$ and $k(3)$.

   $k(1) = |1+2|+3 = \boxed{6}$ $k(3) = -3+4 = \boxed{1}$

   (b) Graph $k(x)$.



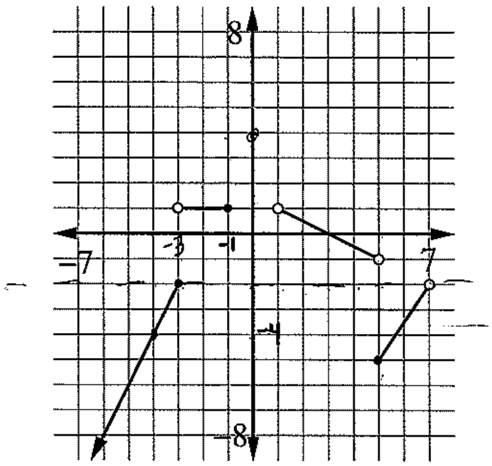
$k(x) = |x+2|+3$
shift $|x|$ left 2 and up 3

$k(x) = -x+4$

"$k(1)$" $= -1+4 = 3$ open circle

$k(2) = -2+4 = 2$

7. Use the function f, whose graph is shown below, to answer each of the following.



(a) Find $f(-4)$, $f(-3)$, $f(1)$, and $f(5)$.

$f(-4) = -4$, $f(-3) = -2$, $f(1)$ does not exist, $f(5) = -5$

(b) Find the values of x such that $f(x) = -2$.

$y = -2$ when $\boxed{x = -3}$.

(c) Find the values of x such that $f(x) < -2$.

$y < -2$ on $(-\infty, -3) \cup [5, 7)$

(d) Find the values of x such that $f(x)$ is positive.

$f(x) > 0$ or $y > 0$ on $\boxed{(-3, -1] \cup (1, 3)}$

(where graph is above the x-axis)

(e) Find the domain and range of $f(x)$.

Domain: $(-\infty, -1] \cup (1, 7)$

Range: $(-\infty, -2] \cup (-1, 1]$

(f) Write a piecewise-defined function for $f(x)$.

$$f(x) = \begin{cases} 2x + 4 & \text{if } -\infty < x \le -3 \\ 1 & \text{if } -3 < x \le -1 \\ -\frac{1}{2}x + \frac{3}{2} & \text{if } 1 < x < 5 \\ \frac{3}{2}x - \frac{25}{2} & \text{if } 5 \le x < 7 \end{cases}$$

(1,1) and (3,0)

$m = \frac{0-1}{3-1} = \frac{-1}{2}$

$y - 0 = -\frac{1}{2}(x - 3)$

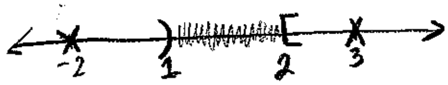
$y = -\frac{1}{2}x + \frac{3}{2}$

(7,−2) and (5,−5)

$m = \frac{-5+2}{5-7} = \frac{-3}{-2} = \frac{3}{2}$

$y + 2 = \frac{3}{2}(x - 7)$

$y = \frac{3}{2}x - \frac{25}{2}$



8. Find the domain of the function $f(x) = \begin{cases} \frac{3x+2}{x^2-4} & \text{if } x < 1 \\ \frac{x^2+x-12}{x-3} & \text{if } x \geq 2 \end{cases}$.

$x^2 - 4 = 0$

$(x+2)(x-2) = 0$

so $x \neq -2, 2$ but only $-2 < 1$

$x - 3 = 0$

so $x \neq 3$

$\boxed{\text{Domain: } (-\infty,-2)\cup(-2,1)\cup[2,3)\cup(3,\infty)}$

9. At an internet cafe, internet users pay a flat fee of \$5 for access, and then additional charges are added based on the number of minutes of internet usage. The first 10 minutes are free, but for the next 30 minutes, a fee of \$0.15 per minute is added. Any minutes used beyond this are charged at a rate of \$0.10 per minute.

(a) Write a piecewise-defined function that gives the amount owed by a person who uses the cafe's internet connection for x minutes.

$$f(x) = \begin{cases} 5 & \text{if } 0 \leq x \leq 10 \\ 5 + 0.15(x-10) & \text{if } 10 < x \leq 40 \\ 9.5 + 0.10(x-40) & \text{if } x > 40 \end{cases}$$

$x > 40$

$5 + 0.15(30) + 0.10(x-40)$

$= 9.5 + 0.10(x-40)$

(b) How much is owed by a customer who used the internet for half an hour?

30min

If $x = 30$, $f(30) = 5 + 0.15(30-10)$

$= \boxed{\$8}$

10. Find the x and y intercepts for each of the following.

(a) $f(x) = 2x^2 + 2x - 24$

y-intercept: Let $x = 0$

$f(0) = -24$

$\boxed{\text{y-int: } (0,-24)}$

x-intercepts: Solve $f(x) = 0$ (if possible)

$2x^2 + 2x - 24 = 0$

$2(x^2 + x - 12) = 0$

$2(x+4)(x-3) = 0$

$x = -4, 3$ so $\boxed{\text{x-intercepts: } (-4,0) \text{ and } (3,0)}$

(b) $g(x) = (x-3)^2 + 5$

$g(0) = (0-3)^2 + 5$

$= 9 + 5$

$= 14$

so $\boxed{(0,14) \text{ is y-intercept.}}$

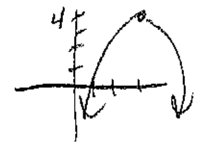
Note: $g(x)$ is a quadratic function in vertex form, so $g(x)$ is a parabola opening upward with vertex $(3,5)$. This parabola will never cross the x-axis, so there are no x-intercepts.

always a y-value

11. For each of the following quadratic functions, find (i) the vertex, (ii) the vertex form of the equation of the quadratic function, (iii) the maximum or minimum value, and (iv) the range.

(a) $f(x) = -x^2 + 6x - 5$

parabola opening downward



By completing the square: $f(x) = -(x^2 - 6x + 5)$

$= -(x^2 - 6x + \underline{9} - \underline{9} + 5)$

$\left(\frac{-6}{2}\right)^2 = (-3)^2 = 9$

$= -((x-3)^2 - 4)$

$\boxed{f(x) = -(x-3)^2 + 4}$ vertex form

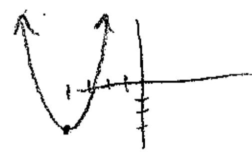
Vertex is $\boxed{(3,4)}$

maximum value $= \boxed{4}$

$\boxed{\text{Range is } (-\infty, 4]}$

(b) $g(x) = 2x^2 + 16x + 29$

$h = $ x-coord. of vertex $= \frac{-b}{2a} = \frac{-16}{2(2)} = -4$



$k = $ y-coord. of vertex $= g(-\frac{b}{2a}) = g(-4) = 2(-4)^2 + 16(-4) + 29 = -3$

$\boxed{\text{vertex} = (-4, -3)}$

Vertex form $= g(x) = a(x-h)^2 + k$

Vertex form: $\boxed{g(x) = 2(x+4)^2 - 3}$

parabola opening upward

Minimum value $= \boxed{-3}$

Range is $[-3, \infty)$

12. Determine which of the following functions are polynomials. For those that are polynomials, describe their end behavior.

non-neg. integer powers on x
coefficients are real numbers

neg. power on x

(a) $y = -\frac{1}{3}x^4 - 2x^{-2} + 5$

not a polynomial

(b) $y = \pi x^5 + 3x^4 - 7x + \frac{1}{\pi}$

polynomial, odd degree, so as $x \to \infty$, $y \to \infty$ and as $x \to -\infty$, $y \to -\infty$ since leading coefficient is positive.

(c) $y = \sqrt{x} + 3x + 7$

$x^{1/2}$ not a polynomial

(d) $y = -3x^2 + \frac{1}{e}x$

polynomial, even degree, neg. leading coefficient so as $x \to \infty$, $y \to -\infty$ and as $x \to -\infty$, $y \to -\infty$

(e) $y = 7x^3 - 3x^2 + 2|x| + 7$

not a polynomial

$p \geq 0$ so

13. Acme Chairs, Inc., has determined the price-demand function for its recliner to be $p = 600 - 5x$ dollars per recliner. Acme has fixed costs that amount to \$6,000 and variable costs of \$130 per recliner.

$600 - 5x \geq 0$
$-5x \geq -600$
$x \leq 120$

(a) Find the price per recliner that maximizes revenue.

find where max revenue occurs first

$R(x) = px = (600-5x)x$

$R(x) = 600x - 5x^2$ or $R(x) = -5x^2 + 600x$

max revenue at $x = -\frac{600}{2(-5)} = 60$

Now find price when $x = 60$:

$p = 600 - 5(60) = \boxed{\$300 \text{ per recliner}}$

(b) What is Acme's maximum profit? How many recliners must be produced and sold to achieve the maximum profit?

$P(x) = R(x) - C(x)$

$= -5x^2 + 600x - (130x + 6000)$

$P(x) = -5x^2 + 470x - 6000$ (a parabola opening downward)

max at $x = \frac{-470}{2(-5)} = 47$ recliners

max Profit $= P(47) = \$5045$

When 47 recliners are produced and sold, Acme will achieve a maximum profit of \$5045.

(c) Find the break-even quantity (or quantities).

$R(x) = C(x)$ or $P(x) = 0$

store in y_1 and find zeros. or use quadratic formula

$-5x^2 + 470x - 6000 = 0$

when $x \approx 15.2352$ and

$x \approx 78.7648$

Acme will (approximately) break even when 15 or 79 recliners are produced and sold.

(d) For what production levels will Acme experience a loss?

$P(x) < 0$

(where $P(x)$ is below x-axis)

$P(x) < 0$ for all integers in the interval $[0, 15] \cup [79, \infty)$