Math 166 - Week in Review #2

Section 1.2 - The Number of Elements in a Set

- The number of elements in a set A is denoted by $n(A)$.
- For any two sets A and B, $n(A \cup B) = n(A) + n(B) - n(A \cap B)$.
- If A and B are mutually exclusive events, then $n(A \cup B) = n(A) + n(B)$.

Section 1.3 - Sample Spaces and Events

- An **experiment** is an activity with observable results (called **outcomes**).
- Sample Space - the set of all possible outcomes of an experiment
- Event - a subset of a sample space of an experiment
- An event E is said to **occur** in a trial of an experiment whenever E contains the observed outcome.
- **Elementary Event** - an event that contains a single outcome.
- **Impossible Event** - The empty set, $\emptyset$, is called the impossible event; it cannot occur since $\emptyset$ has no elements (outcomes).
- **Certainty Event** - If S is the sample space of an experiment and the event E is equal to S, then we say that E is the certainty event.
- Unions, intersections, and complements of events are found in the same ways as they are for sets.
- Two events E and F are **mutually exclusive** if $E \cap F = \emptyset$ (i.e., if it is impossible for both E and F to occur at the same time).
- **Continuous Sample Space** - If the outcome of an experiment can be any real number in an interval of real numbers, then we say that the experiment has a continuous sample space.

Section 1.4 - Basics of Probability

- **Probability of an Event** - The probability of an event is a number between 0 and 1 (inclusive) that measures the likelihood that the event will occur. The closer the probability is to 1, the more likely the event is to occur.
- **Uniform Sample Space** - A sample space in which all outcomes are equally likely is called a uniform sample space.
- **Probability of an Event in a Uniform Sample Space** If S is a finite uniform sample space and E is any event, then the probability of E, denoted $P(E)$, is given by $P(E) = \dfrac{\text{Number of elements in } E}{\text{Number of elements in } S} = \dfrac{n(E)}{n(S)}$.
- **Probability Distribution** - a table that lists events of an experiment and their corresponding probabilities. The events chosen to be in this table must be mutually exclusive, and the probabilities must add to 1.

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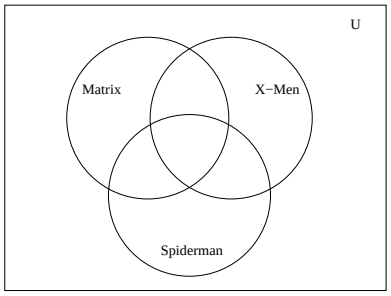
1. Let $A = \{a, c, h, i\}$, $B = \{b, c, f, g\}$, and $C = \{a, b, c, d, e, i\}$.
   (a) Find $n(A)$.
   (b) Find $n(B \cup C)$.
   (c) Find $n(A \cap B)$.

2. A survey of 300 people found that 95 of those surveyed like licorice, 75 like taffy and licorice, and 53 like neither of these two candies.
   (a) How many people surveyed like at least one of the two types of candy?
   (b) How many people surveyed like exactly one of these two types of candy?
   (c) How many people surveyed like taffy but not licorice?
3. One card is selected at random from a standard deck of 52 playing cards, and the suit of the card is recorded. What is the sample space of this experiment?
4. One card is selected at random from a standard deck of 52 playing cards, and the color is recorded. What is the sample space of this experiment?
5. The numbers 0, 1, 2, and 3 are written onto 4 individual slips of paper and placed in a bag. Two numbers are selected at random from the bag and their product is recorded. What is the sample space of this experiment?
6. If $n(C) = 150$, $n(A \cap B \cap C) = 15$, $n(A \cap B^c \cap C) = 35$, and $n(B \cap C) = 70$, find $n(A^c \cap B^c \cap C)$.
7. In a survey of 800 people, it was found that 221 owned a car but not a bicycle, 198 owned a bicycle but not a car, and 105 owned neither a car nor a bicycle.
   (a) How many people surveyed owned both a car and a bicycle?
   (b) How many people surveyed owned a bicycle?
8. An experiment consists of measuring the length (in inches) of fish caught in Random Lake.
   (a) What is the sample space of this experiment?
   (b) Write the event that the fish is at least 10 inches long.
   (c) Write the event that the fish is no more than 7 inches long.
   (d) Write the event that the fish is more than 6 inches but at most 9 inches long.
9. A survey of some college students was conducted to see which of the following three movies they had seen: *The Matrix*, *X-Men*, and *Spiderman*. It was found that
   - 6 students had seen all three movies.
   - 8 students had seen *The Matrix* and *X-Men*
   - 3 students had seen *X-Men* and *Spiderman* but not *The Matrix*.
   - 6 students had seen exactly 2 of the 3 movies.
   - 10 students had seen neither *X-Men* nor *The Matrix*.
   - 19 students had seen *The Matrix*.
   - 26 students had seen *The Matrix* or *X-Men*.
   - 22 students had seen *X-Men* or *Spiderman*.

   (a) Fill in the Venn Diagram, illustrating the above information.



(b) How many students surveyed had seen at least one of the three movies?

(c) How many students surveyed had seen only *Spiderman*?

(d) How many students surveyed had seen *The Matrix* or *X-Men* but not both?

(e) How many students surveyed had seen *The Matrix* and *Spiderman*?

10. Some students were asked whether they had one or more of the following types of animals as children: dog, cat, fish.

39	said they did not have a dog.
28	said they only had a dog.
16	said they had a dog and a fish.
47	said they had a fish or a cat.
15	said they had a fish but did not have a cat.
48	said they only had one of these types of pets.
6	said they had all three of these pets.
16	said they did not have a dog or a cat.

(a) Fill in a Venn Diagram illustrating the above information.

(b) How many students were in the survey?

11. Let E and F be two events of an experiment such that $P(E) = 0.35$, $P(F) = 0.4$, and $P(E \cup F) = 0.62$. Find

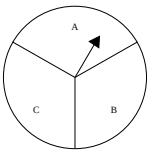
(a) $P(E \cap F)$

(b) $P(E^c \cup F^c)$

(c) $P(E \cap F^c)$

(d) $P(E \cup F^c)$

12. A fair coin is tossed. If the coin lands heads, then the hand on the spinner below is spun. (This spinner never lands on a line.) If the coin lands tails, then a fair die is rolled and the number landing up is recorded.



(a) What is the sample space of this experiment?

(b) Write the event that an odd number is rolled.

(c) Write the event that the coin lands heads.

13. An experiment consists of randomly selecting an integer multiple of 3 that is between 3 and 21 (inclusive).

(a) What is the sample space of this experiment?

(b) Write the event E that the number selected is even.

(c) Write the event F that the number selected is a multiple of 4.

(d) Write the event G that the number selected is odd and less than 15.

(e) Which pairs of the events E, F, and G are mutually exclusive?

(f) If the number selected was 12, which of the events E, F, and G have occurred?

14. In a survey, respondents are asked two questions dealing with their level of satisfaction with their electric company. For each question, the response options are “Happy,” “Unhappy,” and “Neither.”

(a) What is the sample space of this experiment?

(b) Write the event E that at least one question was answered as "Neither."

(c) Write the event F that the second question was answered as "Happy."

(d) Are E and F mutually exclusive events?

15. One card is selected at random from a standard deck of 52 cards. What is the probability that the card is

(a) a nine.

(b) a diamond.

(c) a red face card.

16. An experiment was conducted to find out how long customers must wait on hold while trying to call a customer service representative of a certain company. It was found that 17% of customers wait less than 5 minutes, 53% of customers wait less than 10 minutes, and 12% of customers wait 15 minutes or more.

(a) Organize this information in a probability distribution.

(b) What is the probability that a randomly selected caller waits at least 10 minutes?

17. The students in one section of Math 166 were asked how many credit hours they are currently taking. The table summarizes their responses:

Number of Hours	3	6	9	12	13	14	15	16	17	18	19
Number of Students	4	2	5	14	17	19	20	13	9	2	1

(a) What is the empirical probability that a randomly selected student from this class is taking 15 hours?

(b) What is the empirical probability that a randomly selected student from this class is taking less than 12 hours?

(c) What is the empirical probability that a randomly selected student from this class is taking at least 14 hours?