Math 142 - Week in Review #3

1. Simplify each of the following if possible.

(a)
$$\left(\frac{4e^{-7x}}{e^{x-4}}\right)^3$$

(b)
$$9^{3x+1} \cdot 27^{2-x}$$

(c)
$$5^x + 5^{2x}$$

(d)
$$2^{\log_2 7}$$

(e)
$$9^{\log_3 x}$$

(f)
$$e^{-2\ln x}$$

2. Solve each of the following for x.

(a)
$$5^{11x-4} = 125^{x^2+2}$$

(b)
$$2^x \cdot 7x + 2^{x+1} = 0$$

(c)
$$2^x \cdot 8^{x+3} = 32$$

(d)
$$e^{2x} + 3e^x = 40$$

(e)
$$7^x + 49^x = 20$$

(f)
$$3^{x^2+2x} = \frac{1}{27^x}$$

3. Find the domain of each of the following.

(a)
$$f(x) = 7 \cdot 3^{2x-5}$$

(b)
$$g(x) = \log_5(x+2)$$

(c)
$$m(x) = \ln |x|$$

(d)
$$k(x) = \frac{\sqrt{2x+10}}{\log_2(x+3)}$$
 (Hint: $\log_b 1 = 0$)

- 4. Bob invested \$4,100 in an account paying 3.25% per year compounded quarterly.
 - (a) How much money will Bob have in his account in 10 years?

(b) How long will it take for Bob's account to reach \$10,000?

5. Rewrite each of the following in exponential form and solve for x, y, or b as indicated.

(a)
$$\log_7 x = -2$$

(b)
$$\log(x-3) = 3$$

(c)
$$\log_{16}\left(\frac{1}{4}\right) = y$$

(d)
$$\log_b 125 = 3$$

(e)
$$\log_b 9 = \frac{2}{3}$$

6. The table below shows the relationship between dollars spent on advertising and the number of units of an item sold.

| Dollars spent | 5,000 | 10,000 | 25,000 | 30,000 | 40,000 | 48,000 |
|------------------------|-------|--------|--------|--------|--------|--------|
| Millions of units sold | 8.534 | 8.757 | 9.050 | 9.109 | 9.201 | 9.260 |

- (a) Use your calculator to view a scatter plot of this data set. What types of functions should NOT be considered as a model for this data set? Justify your answer.
- (b) If your goal is to find a model that can be use for predicting the number of units sold when more and more money is used for advertising, which type of function seems most appropriate to model the data? Find this function and write it as a model.
- (c) Use the model to estimate the number of units sold if \$82,000 is spent on advertising.

7. Write each of the following as a sum or difference of logarithms.

(a)
$$\log_5 \frac{25x^2y}{w^3z}$$

(b)
$$\ln \frac{\sqrt{3x^2+7}}{x^5(x-7)^2}$$

8. Solve for *x* in each of the following.

(a)
$$\log x + \log (x+3) = \log 18$$

(b)
$$\log_7 x = 4\log_7 2 - \frac{1}{3}\log_7 8$$

9. Fred invested some inheritance money in a mutual fund that pays 7.3% per year compounded continuously. How long will it take for his investment to double?

- 10. The number of bacteria in a culture after t days can be modeled by $A(t) = 13e^{0.043t}$ thousand bacteria.
 - (a) What is the size of the initial population?
 - (b) How many bacterial will be in the culture after 18 hours?

(c) How long will it take for the population of bacteria to grow to 125,000?

11. A student was given a vitamin pill. The concentration of the vitamin in the student's body at different times is given in the table below.

| | | | 11:30am | 2:15pm | 3pm | 4:30pm |
|------------------------------------|-----|-----|---------|--------|-----|--------|
| Concentration $(\frac{\mu g}{mL})$ | 5.2 | 3.0 | 2.0 | 0.9 | 0.7 | 0.5 |

- (a) Find the best model for the data where t is the number of hours since 6am.
- (b) According to your model, when will the concentration reach 0.25 $\frac{\mu g}{mL}$?

12. Which of the following functions are one-to-one?

(a)
$$f(x) = 3 | x - 4 |$$

(b)
$$h(x) = x - 3$$

(c)
$$g(x) = e^{x+3} + 4$$

$$(d) \ m(x) = \ln(x-1)$$

13. Find the inverse of each of the following functions.

$$(a) f(x) = \ln(x+3)$$

(b)
$$g(x) = 5 \cdot 7^x$$