Math 166 - Week in Review #3 - Exam 1 Review

NOTE: For reviews of the other sections on Exam 1, refer to the first page of WIR #1 and #2.

Section 2.6 - The Inverse of a Square Matrix

- Only square matrices can have inverses, *but not all square matrices have inverses.*
- A square matrix that does not have an inverse is called a **singular matrix**.
- The inverse of A, denoted A^{-1}, is the square $n \times n$ matrix such that $AA^{-1} = A^{-1}A = I_n$.
- Systems of equations can be represented as a matrix equation of the form $AX = B$ where A is the coefficient matrix, X is a column vector containing the variables, and B is a column vector containing constants.
- If A has an inverse, the solution to the matrix equation is $X = A^{-1}B$.
- If A does not have an inverse (i.e., if A is singular), this does NOT imply the system has no solution. It simply means that you must use another method to solve the system.

Section 2.7 - Leontief Input-Output Model - *Not all instructors covered this topic.*

- The input-output model is used to study the relationship between industrial production and consumer demand.
- If A is the input-output matrix and X represents the total output of all industries, then AX represents the internal consumption for that economy.
- Subtracting the internal consumption from the total output gives the net output of goods and services for consumer demand, D:

$$X - AX = D$$

- To find the amount of goods and services that must be produced to satisfy consumer demand, solve for X in the above equation to obtain

$$X = (I - A)^{-1}D$$

Chapter 5 - Finance

- **Simple Interest** - interest that is computed on the original principal only
- Simple Interest Formulas

  Interest $= I = Prt$

  Accumulated Amount $= A = P + I = P + Prt = P(1 + rt)$

  NOTATION: I = interest earned, P = principal, r = interest rate (as a decimal), t = term of the investment in ***YEARS***, A = accumulated amount

- **The TVM-Solver CANNOT be used for simple interest calculations.**
- **Compound Interest** - earned interest that is periodically added to the principal and thereafter itself earns interest at the same rate.
- The TVM-Solver can be used in problems involving compound interest as follows:

N	=	total number of payments made, usually $m \times t$.
I%	=	interest rate in *percent form*. Don't convert to decimal form!!
PV	=	present value (prinicpal, or the amount you start with). Entered as negative if invested, positive if borrowed.
PMT	=	payment (amount paid each period). Entered as negative if paying off a loan, positive if receiving money, 0 if computing compound interest.
FV	=	future value (accumulated amount). This will be 0 if paying off a loan.
P/Y	=	number of payments per year (usually the same as m).
C/Y	=	number of conversions per year (m).

spring 2007

- At the bottom of the screen, you will see PMT:END BEGIN. If END is highlighted, then the TVM Solver calculates everything with payments being made at the end of the period. For virtually all of the problems we will work in class, END should be highlighted.
- You can solve for any quantity on the TVM-Solver by moving the cursor to that quantity and then pressing ALPHA followed by ENTER.
- **Continuously Compounded Interest:** $A = Pe^{rt}$
- **Effective Rate of Interest** - The effective rate of interest is a way of comparing interest rates. More precisely, the *effective rate* is the simple interest rate that would produce the same accumulated amount in 1 year as the nominal rate compounded m times per year.
- The effective rate of interest is typically denoted by r_{eff} and is also known as the effective annual yield.
- To calculate the effective rate of interest, use the Eff( ) function on the calculator. This function can be found under Finance–just arrow down until you see C: Eff(.
- The Eff( ) function has two parameters, the nominal (or annual) interest rate entered as a percent, and the number of conversion, m, per year: Eff(nominal rate as a percent, m)
- **Annuity** - a sequence of payments made at regular time intervals.
- In this course, we will study annuities with the following properties:
  1. The terms are given by fixed time intervals.
  2. The periodic payments are equal in size.
  3. The payments are made at the end of the payment periods.
  4. The payment periods coincide with the interest conversion periods.

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1. True/False

TRUE FALSE a) $I_nA = AI_n = A$ for all matrices A. Would be true if we knew A was $n \times n$

TRUE FALSE b) A nonsingular matrix has no inverse.

singular → no inverse

TRUE FALSE c) To be able to compute the matrix product AB, the number of columns of A must equal the number of rows of B.

TRUE FALSE d) If B is a 2×2 matrix, then $B + I_2 = B$. $BI_2 = B$

TRUE FALSE e) When solving the matrix equation $AX = B$ by computing $A^{-1}B$ in the calculator, a message of "ERR: SINGULAR MAT" implies that the system of equations has no solution. means there is not a unique soln. Could be inf. many or no soln.

TRUE FALSE f) Every square matrix has an inverse.

TRUE FALSE g) If the parametric solution to a system of equations is $(3t+2, -t-3, t)$, then $(-7, 0, -3)$ is a particular solution.

$t = -3$

$x = 3(-3) + 2 = -7$

$y = -(-3) - 3 = 0$

parallel lines → same slope, dif. y-int

2. Find the value of k so that the following system has no solution:
$$\begin{aligned} 5x+2y &= -7 \\ -3x+ky &= 8 \end{aligned}$$

$5x+2y=-7$

$2y=-5x-7$

$y=\frac{-5}{2}x-\frac{7}{2}$

$m=-\frac{5}{2}$

$-3x+ky=8$

$ky=3x+8$

$y=\frac{3}{k}x+\frac{8}{k}$

$m=\frac{3}{k}$

$-\frac{5}{2}=\frac{3}{k}$

$-5k=6$

$k=\boxed{-\frac{6}{5}}$

Verify

$$\left[\begin{array}{cc|c} 5 & 2 & -7 \\ -3 & -6/5 & 8 \end{array}\right] \xrightarrow{rref} \left[\begin{array}{cc|c} 1 & -4 & 0 \\ 0 & 0 & 1 \end{array}\right]$$

No solution

Another way to verify: check y-ints

$b_1=-\frac{7}{2}$

$b_2=8/k=8/-\frac{6}{5}$

$=\frac{8}{6}\cdot-\frac{5}{6}$

$=-\frac{20}{3}\neq b_1$

3. For each of the following, if the matrix is in row reduced form, interpret its meaning as a solution to a system of equations. If the matrix is not in row reduced form, explain why.

(a) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & \cancel{3} & -5 \\ 0 & 0 & 1 & 0 \end{array}\right]$ Not in row-reduced form. Column 3 has a leading 1 but not all other entries in that column are 0.

(b) $\begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \left[\begin{array}{ccc|c} 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$ Is in row-reduced form.

Let $x=t$, where t is any real #. $(t,-3,8)$

$y=-3$
$z=8$

(c) $\begin{array}{c} \begin{array}{cc} x & y \end{array} \\ \left[\begin{array}{cc|c} 1 & 0 & 15 \\ 0 & 1 & -7 \\ 0 & 0 & 0 \end{array}\right] \end{array}$ $x=15$, $y=-7$ $\boxed{(15,-7)}$

Is in row-red. form

(d) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & -1 & 0 & -3 \\ 0 & 0 & 1 & 8 \end{array}\right]$ Not in row-reduced form. The 1st nonzero entry in row 2 is not 1.

(e) $\left[\begin{array}{ccc|c} 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 1 & 0 & 0 & 1 \end{array}\right]$ Not in row-reduced form. Leading 1's are not down and to the right of each other.

(f) $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{array}\right]$ Is in row-reduced form.

$\boxed{\text{No solution}}$

(g) $\left[\begin{array}{cc|c} 0 & 0 & 0 \\ 1 & 0 & 3 \\ 0 & 1 & 8 \end{array}\right]$ Not in row-reduced form. Rows of all 0's must be below all other rows containing nonzero entries.

4. For the next two word problems do the following:
   I) Define the variables that are used in setting up the system of equations.
   II) Set up the system of equations that represents this problem. III) Solve for the solution.
   IV) If the solution is parametric, then tell what restrictions should be placed on the parameter(s). Also give three specic solutions.

(a) Fred, Bob, and George are avid collectors of baseball cards. Among the three of them, they have 924 cards. Bob has three times as many cards as Fred, and George has 100 more cards than Fred and Bob do combined. How many cards do each of the friends have?

Let x = the number of cards Fred has.
Let y = the number of cards Bob has.
Let z = the number of cards George has.

$x = 103$
$y = 309$
$z = 512$

System:

$$\begin{aligned} x+y+z &= 924 \\ y &= 3x \\ z &= x+y+100 \end{aligned} \rightarrow \begin{aligned} x+y+z &= 924 \\ -3x+y &= 0 \\ -x-y+z &= 100 \end{aligned} \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 924 \\ -3 & 1 & 0 & 0 \\ -1 & -1 & 1 & 100 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 103 \\ 0 & 1 & 0 & 309 \\ 0 & 0 & 1 & 512 \end{array}\right]$$

Fred has 103 cards, Bob has 309 cards, and George has 512 cards.

(b) In a laboratory experiment, a researcher wants to provide a rabbit with exactly 1000 units of vitamin A, exactly 1600 units of vitamin C and exactly 2400 units of vitamin E. The rabbit is fed a mixture of three foods. Each gram of food 1 contains 2 units of vitamin A, 3 units of vitamin C, and 5 units of vitamin E. Each gram of food 2 contains 4 units of vitamin A, 7 units of vitamin C, and 9 units of vitamin E. Each gram of food 3 contains 6 units of vitamin A, 10 units of vitamin C, and 14 units of vitamin E. How many grams of each food should the rabbit be fed?

Let x = the # of grams of food 1.
Let y = the # of grams of food 2.
Let z = the # of grams of food 3.

	Food 1	Food 2	Food 3	Exact Requirement
units of Vit A per gram	2	4	6	1000
units of Vit C per gram	3	7	10	1600
units of Vit E per gram	5	9	14	2400

System:

$$\begin{aligned} 2x+4y+6z &= 1000 \\ 3x+7y+10z &= 1600 \\ 5x+9y+14z &= 2400 \end{aligned} \rightarrow \left[\begin{array}{ccc|c} 2 & 4 & 6 & 1000 \\ 3 & 7 & 10 & 1600 \\ 5 & 9 & 14 & 2400 \end{array}\right] \xrightarrow{\text{rref}} \begin{array}{c} \begin{array}{ccc} x & y & z \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & 1 & 300 \\ 0 & 1 & 1 & 100 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array}$$

Let $z = t$, where t is any number greater than or equal to 0.

$$\begin{aligned} x+z &= 300 \\ x &= -z+300 \\ x &= -t+300 \geq 0 \\ -t+300 &\geq 0 \\ -t &\geq -300 \\ t &\leq 300 \end{aligned} \qquad \begin{aligned} y+z &= 100 \\ y &= -z+100 \\ y &= -t+100 \geq 0 \\ -t+100 &\geq 0 \\ -t &\geq -100 \\ t &\leq 100 \end{aligned}$$

Parametric Solution: $(-t+300, -t+100, t)$, where $0 \leq t \leq 100$

$-t+300$: # of grams of food 1; $-t+100$: # of grams of food 2; t: # of grams of food 3.

If $t=0$,
$x = 0+300 = 300$ grams of food 1
$y = 0+100 = 100$ grams of food 2
$z = 0$ grams of food 3

If $t=50$,
$x = -50+300 = 250$ grams of food 1
$y = -50+100 = 50$ grams of food 2
$z = 50$ grams of food 3.

If $t=100$,
$x = -100+300 = 200$ grams of food 1
$y = -100+100 = 0$ grams of food 2
$z = 100$ grams of food 3.

5. Solve the system $\begin{aligned} 5x-3y &= 7 \\ 2x+6y &= -1 \end{aligned}$ using the Gauss-Jordan elimination method.

$$\left[\begin{array}{cc|c} 5 & -3 & 7 \\ 2 & 6 & -1 \end{array}\right] \xrightarrow{R_1-2R_2 \to R_1} \left[\begin{array}{cc|c} 1 & -15 & 9 \\ 2 & 6 & -1 \end{array}\right] \xrightarrow{R_2-2R_1 \to R_2} \left[\begin{array}{cc|c} 1 & -15 & 9 \\ 0 & 36 & -19 \end{array}\right]$$

$$[5\ -3\ 7] - 2[2\ 6\ -1]$$
$$= [5\ -3\ 7] + [-4\ -12\ 2]$$
$$= [1\ -15\ 9]$$

$$[2\ 6\ -1] - 2[1\ -15\ 9]$$
$$= [2\ 6\ -1] + [-2\ 30\ -18]$$
$$= [0\ 36\ -19]$$

$$\xrightarrow{\frac{1}{36}R_2} \left[\begin{array}{cc|c} 1 & -15 & 9 \\ 0 & 1 & -\frac{19}{36} \end{array}\right] \xrightarrow{R_1+15R_2} \left[\begin{array}{cc|c} 1 & 0 & \frac{13}{12} \\ 0 & 1 & -\frac{19}{36} \end{array}\right] \quad \boxed{\begin{aligned} x &= \frac{13}{12} \\ y &= -\frac{19}{36} \end{aligned}}$$

$$[1\ -15\ 9] + \cancel{15}^{5}[0\ 1\ -\tfrac{19}{\cancel{36}_{12}}]$$
$$= [1\ -15\ 9] + [0\ 15\ -\tfrac{95}{12}] = [1\ 0\ \tfrac{13}{12}]$$

6. Solve for the variables x, y, z, and u. If this is not possible, explain why.

$$\begin{bmatrix} 2 & 0 \\ x & -1 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ y+1 & z \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ z & x \end{bmatrix}^T = -2\begin{bmatrix} 1-u & 5 \\ -3 & -\frac{3}{2} \end{bmatrix}$$

$$\begin{bmatrix} 2 & 8 \\ x-1(y+1) & 4x-z \end{bmatrix} + \begin{bmatrix} 7 & z \\ 0 & x \end{bmatrix} = \begin{bmatrix} 2u-2 & -10 \\ 6 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 9 & 8+z \\ x-y-1 & 5x-z \end{bmatrix} = \begin{bmatrix} 2u-2 & -10 \\ 6 & 3 \end{bmatrix}$$

$2u-2=9$
$2u=11$
$\boxed{u=\frac{11}{2}}$

$8+z=-10$
$\boxed{z=-18}$

$5x-z=3$
$5x+18=3$
$5x=-15$
$\boxed{x=-3}$

$x-y-1=6$
$-3-y-1=6$
$-y=10$
$\boxed{y=-10}$

7. Using matrix algebra, solve for the matrix D: (Assume that inverses exist where needed and all sizes are appropriate.)

$$D = AD + B$$

$$D - AD = B$$

$$(I_n - A)^{-1}(I_n - A)D \overset{-1}{=} B$$

$$D = (I_n - A)^{-1} B$$

8. Find the matrix A that makes the following equation true:

$$\begin{bmatrix} -5 & 3 \\ 8 & 7 \end{bmatrix} + \frac{1}{3}\begin{bmatrix} 2 & -1 \\ -7 & 5 \end{bmatrix} A = I_2$$

$$\frac{1}{3}\begin{bmatrix} 2 & -1 \\ -7 & 5 \end{bmatrix} A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -5 & 3 \\ 8 & 7 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ -7 & 5 \end{bmatrix}^{-1}\begin{bmatrix} 2 & -1 \\ -7 & 5 \end{bmatrix} A = (\tfrac{1}{3})^{-1}\begin{bmatrix} 6 & -3 \\ -8 & -6 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & -1 \\ -7 & 5 \end{bmatrix}^{-1}\begin{bmatrix} 18 & -9 \\ -24 & -18 \end{bmatrix}$$

$$A = \begin{bmatrix} 22 & -21 \\ 26 & -33 \end{bmatrix}$$

ANS: $A = \begin{bmatrix} 22 & -21 \\ 26 & -33 \end{bmatrix}$

9. Use the given matrices to compute each of the following. If an operation is not possible, explain why.

$$A = \begin{bmatrix} -5 & 3 \\ 7 & 8 \end{bmatrix},\ B = \begin{bmatrix} 1 & 2 \\ 5 & -8 \\ 3 & 5 \end{bmatrix},\ C = \begin{bmatrix} 5 & 4 & 7 \\ 6 & -3 & 1 \end{bmatrix},$$

$D = 3 \times 3$ nonsingular matrix.
$E = 4 \times 4$ singular matrix.

(a) $B + C$

Not possible.
B and C must be the same size to be able to add.

(b) BC

Size of B: 3×2 Size of C: 2×3 ($2 = 2$) → 3×3.

BC = (in calc)

$$\boxed{\begin{bmatrix} 17 & -2 & 9 \\ -23 & 44 & 27 \\ 45 & -3 & 26 \end{bmatrix}}$$

(c) $D^{-1}C$

D^{-1}: 3×3 C: 2×3 ($3 \neq 2$)

Multiplication is not possible because the number of columns of D^{-1} does not equal the number of rows of C.

(d) $AB^T - 5C$

$$\underbrace{\underset{2\times2}{\text{Size of } A}\ \underset{2\times3}{\text{Size of } B^T}}_{(2\times3)\ \checkmark} - \underbrace{\underset{}{\text{size of } C}}_{(2\times3)\ \checkmark}$$

$AB^T - 5C$ (in calc) $= \boxed{\begin{bmatrix} -24 & -69 & -35 \\ -7 & -14 & 56 \end{bmatrix}}$

(e) DD^{-1}

nonsingular = not singular = <u>has</u> an inverse

$DD^{-1} = I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (the 3x3 identity matrix)

(f) $E^{-1}E$

Not possible. E has no inverse.

(singular $\Rightarrow$ no inverse)

(g) CA

Size of C 2×3 $\neq$ Size of A 2×2

Not possible. The number of columns of C does not equal the number of rows of A.

(h) C^{-1}

Not possible. Matrices that are not square cannot have an inverse.

(NOTE: A matrix must be square to have an inverse, but not all square matrices do have inverses.)

10. Work #41 on page 130 of Section 2.5 in your textbook by Tan. This is a problem about understanding the meaning of the entries in the product of two matrices. There is also another example in Week in Review #2.

11. First write the following system as a matrix equation and then solve by using a matrix inverse.

$$\begin{aligned} -3x &= 4y+z \\ y-7 &= -x+5z \\ 2x+z &= 14-y \end{aligned}$$

$$\left.\begin{aligned} -3x-4y-z&=0 \\ x+y-5z&=7 \\ 2x+y+z&=14 \end{aligned}\right\} \text{equivalent to } AX=B \text{ where}$$

$$A=\begin{bmatrix} -3 & -4 & -1 \\ 1 & 1 & -5 \\ 2 & 1 & 1 \end{bmatrix},\ X=\begin{bmatrix} x \\ y \\ z \end{bmatrix},\ B=\begin{bmatrix} 0 \\ 7 \\ 14 \end{bmatrix}.\quad X=\underset{\text{(in calc)}}{A^{-1}B}=\begin{bmatrix} 35/3 \\ -77/9 \\ -7/9 \end{bmatrix}$$

$$\boxed{x=\frac{35}{3},\ y=-\frac{77}{9},\ z=-\frac{7}{9}}$$

12. A simple economy depends on three commodities: oil, corn, and coffee. Production of 1 unit of oil requires 0.1 units of oil and 0.2 units of corn. Producing 1 unit of corn requires 0.2 units of oil, 0.1 units of corn, and 0.05 units of coffee. To produce 1 unit of coffee, 0.1 units of oil, 0.05 units of corn, and 0.1 units of coffee are used.

(a) How many units of oil are consumed in the production of 35 units of coffee?

$$(0.1)(35)=\boxed{3.5 \text{ units}}$$

(b) Find the production level required to meet an external demand of 1,000 units of each of these three commodities. (#18, pg. 114 of *Finite Mathematics and Calculus* by Lial, Greenwell, and Ritchey)

Input-output matrix

output

$$A=\begin{array}{c|c} & \begin{array}{ccc} \text{oil} & \text{corn} & \text{coffee} \end{array} \\ \begin{array}{c} \text{oil} \\ \text{corn} \\ \text{coffee} \end{array} & \begin{bmatrix} 0.1 & 0.2 & 0.1 \\ 0.2 & 0.1 & 0.05 \\ 0 & 0.05 & 0.1 \end{bmatrix} \end{array},\quad \text{Demand } D=\begin{bmatrix} 1000 \\ 1000 \\ 1000 \end{bmatrix}$$

Input

$$X=(I_3-A)^{-1}D$$

$$X=\begin{bmatrix} 1583.9072 \\ 1529.5397 \\ 1196.0855 \end{bmatrix}$$

The economy must produce 1583.9072 units of oil, 1529.5397 units of corn, and 1196.0855 units of coffee to meet the external demand.

(c) How much of each commodity is consumed internally to meet this demand?

Two ways to find internal consumption:

$$\underset{\text{Total production} - \text{demand}}{X-D}=\begin{bmatrix} 583.9072 \\ 529.5397 \\ 196.0855 \end{bmatrix} \quad \text{or} \quad AX=\begin{bmatrix} 583.9072 \\ 529.5397 \\ 196.0855 \end{bmatrix}$$

583.9072 units of oil, 529.5397 units of corn, and 196.0855 units of coffee are consumed internally.

13. When Erica graduated from high school, she received \$500 from her parents as a gift. She then loaned this money to her brother who repaid her 3 months later with a sum of \$510.25. What was the simple interest rate that Erica charged her brother?

$$I=Prt$$

$$10.25=500r\left(\frac{3}{12}\right)$$

$$\boxed{0.082=r}$$

(8.2% simple interest)

14. Annette wants to take a trip to Europe when she graduates. She will need $4,500 for this trip. How much money should Annette deposit now into an account paying 8%/year compounded quarterly if she expects to graduate in 4 years? How much interest will she earn?

N=4*4 PMT=0
I%=8 FV=4500
PV=? P/Y=C/Y=4

Annette should deposit $3278.01.

Interest earned: 4500-3278.01
= $1,221.99

15. Lynnette, Annette's twin sister, wants to take that same trip to Europe, but she does not have enough money to open the same type of account as Annette. Instead, she plans to make monthly payments to an account paying 8.25%/year compounded monthly. How much should each payment be so that she has $4,500 at the end of 4 years? How much interest will Lynnette earn?

N=12*4 PMT=? → Lynnette should deposit $79.45 each month.
I%=8.25 FV=4500
PV=0 P/Y=C/Y=12

Interest earned: 4500 − 79.45*12*4 (Lynnette's total deposit)
= 4500 − 3813.60
= $686.40

16. If Bank A has a savings account paying 8%/year compounded semiannually and Bank B offers 7.9%/year compounded monthly, which is the better offer?

Bank A: Eff(8, 2) = 8.16% (as a simple interest rate)

Bank B: Eff(7.9, 12) = 8.1924% (as a simple interest rate)

Bank B has the better offer.

17. Juanita decided to purchase a flat-screen HDTV. She makes a down payment of $250 and secures financing for the balance of the purchase price at a rate of 12%/year compounded monthly. Under the terms of the finance agreement, she is required to make monthly payments of $125 for 30 months.

(a) What was the cash price of the TV? = Down payment + amt financed.

Find Amt financed:

N=30 PMT=-125
I%=12 FV=0
PV=? P/Y=C/Y=12

PV=$3225.96

Price of TV = 250 + 3225.96
= $3,475.96

(b) How much interest did Juanita pay?

Total amt paid for TV:

Down payment + monthly payments.

= 250 + 125*30

= 4000

Interest paid: 4000 − 3475.96 = $524.04

18. What is the present value of a sum of \$5,000 due in 6 years at an interest rate of 6.75%/year compounded continuously?

For continuously compounded interest,

$A = Pe^{rt}$

$5000 = Pe^{(.0675*6)} \rightarrow P = 5000/e^{(.0675*6)} = \boxed{\$3,334.88}$

19. Julian opened an account with \$8,000 and after 7 years, it had grown to \$10,000. What was the annual interest rate if interest was compounded weekly?

N=52*7 PMT=0
I%=? FV=10000
PV=-8000 P/Y=C/Y=52

Interest rate: $\boxed{3.1887\%}$

20. Miles and Keiko plan to make a \$25,000 down payment on a \$110,000 home and finance the balance with a 25-year note that has an interest rate of 8.25% compounded monthly.

(a) What monthly payments should they make to pay off the note in 25 years?

N=12*25 PMT=?
I%=8.25 FV=0
PV=85000 P/Y=C/Y=12

110000
-25000
85000

$\boxed{\text{Monthly payments of \$670.18}}$

(b) How much interest will they pay?

Total amt paid for house: 25000 + 670.18*12*25 = \$226,054

Interest paid: 226054 - 110000 = $\boxed{\$116,054}$

(c) How much equity will they have in their home after 10 years of payments?

Equity = Value of house - what you still owe

Find what you owe: N=10*12 PMT=-670.18
I%=8.25 FV=? ⟶ Still owe \$69081.48
PV=85000 P/Y=C/Y=12

Equity =
110000-69081.48
= $\boxed{\$40,918.52}$

(d) Create an amortization schedule for the first 4 months of the loan.

period	interest owed	payment	amount toward principal	outstanding principal
0	——	——	——	85000
1	584.38	670.18	85.80	84914.20
2	583.79	670.18	86.39	84827.81
3	583.19	670.18	86.99	84740.82
4	582.59	670.18	87.59	84653.23

85000*.0825/12

New outstanding principle = old outstanding principle - amt toward principle

= payment - interest owed

In general,
Interest owed = current outstanding principle * interest rate / 12