- If the objective function is optimized at two adjacent vertices of S, then it is optimized at every point on the line segment joining these vertices. In this case, there are infinitely many solutions to the problem.

- **<u>The Method of Corners for a *BOUNDED* Feasible Set</u>**

  Step 1: Graph the feasible set.

  Step 2: Find the coordinates of all corner points of the feasible set.

  Step 3: Evaluate the objective function at each corner point.

  Step 4: Find the vertex which gives the maximum (minimum) value of the objective function.

  - If there is only one such vertex, then this vertex is the unique solution to the problem.
  - If the objective function is maximized (minimized) at two adjacent corner points of S, then there are infinitely many solutions given by the points on the line segment joining these two vertices.

- NOTE: For an *UNBOUNDED* solution set, the procedure for the Method of Corners is the same, but depending on the coefficients of your objective function, a maximum value or a minimum value of the objective function may not be possible.

---

1. True/False

TRUE FALSE a) $I_nA = AI_n = A$ for all matrices A. (This is true for all square matrices A.)

TRUE FALSE b) A nonsingular matrix has no inverse.

TRUE FALSE c) To be able to compute the matrix product AB, the number of columns of A must equal the number of rows of B.

TRUE FALSE d) If B is a 2×2 matrix, then $B + I_2 = B$. $(BI_2 = B$ is true$)$

TRUE FALSE e) When solving the matrix equation $AX = B$ by computing $A^{-1}B$ in the calculator, a message of "ERR: SINGULAR MAT" implies that the system of equations has no solution. (It only implies the solution is not unique)

TRUE FALSE f) Every square matrix has an inverse.

TRUE FALSE g) If the parametric solution to a system of equations is $(3t+2, -t-3, t)$, then $(-7, 0, -3)$ is a particular solution.

$-(-3)-3 = 0$

$3(-3)+2 = -9+2 = -7$

2. Find the value of k so that the following system has no solution: $\begin{aligned} 5x+2y &= -7 \\ -3x+ky &= 8 \end{aligned}$

$5x+2y=-7$

$2y=-5x-7$

$y=-\frac{5}{2}x-\frac{7}{2} \Rightarrow m=-\frac{5}{2}$

$-3x+ky=8$

$ky=3x+8$

$y=\frac{3}{k}x+\frac{8}{k}$

$m=\frac{3}{k}$

$-\frac{5}{2}=\frac{3}{k}$

$\boxed{\begin{aligned}-5k&=6\\ k&=-\frac{6}{5}\end{aligned}}$

Verify

$\left[\begin{array}{cc|c} 5 & 2 & -7 \\ -3 & -\frac{6}{5} & 8 \end{array}\right] \xrightarrow{\text{rref}}$

$\left[\begin{array}{cc|c} 1 & .4 & 0 \\ 0 & 0 & 1 \end{array}\right]$

No soln.

3. For each of the following, if the matrix is in row reduced form, interpret its meaning as a solution to a system of equations. If the matrix is not in row reduced form, explain why.

(a) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 4 \\ 0 & 1 & \cancel{3} & -5 \\ 0 & 0 & 1 & 0 \end{array}\right]$

(b) $\left[\begin{array}{ccc|c} 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 0 \end{array}\right]$

(c) $\left[\begin{array}{cc|c} 1 & 0 & 15 \\ 0 & 1 & -7 \\ 0 & 0 & 0 \end{array}\right]$

(d) $\left[\begin{array}{ccc|c} 1 & 0 & 0 & 5 \\ 0 & -1 & 0 & -3 \\ 0 & 0 & 1 & 8 \end{array}\right]$

(e) $\left[\begin{array}{ccc|c} 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 1 & 0 & 0 & 1 \end{array}\right]$

(f) $\left[\begin{array}{ccc|c} 1 & 1 & 0 & 5 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \end{array}\right]$

(g) $\left[\begin{array}{cc|c} 0 & 0 & 0 \\ 1 & 0 & 3 \\ 0 & 1 & 8 \end{array}\right]$

a) Not in r.r. form b/c col 3 has a leading 1, but not all other entries in col 3 are 0.

b) Is in r.r. form. Let $x=t$ where t is any real #.
$y=-3$, $z=8$ $\boxed{(t,-3,8)}$ = parametric soln

c) Is in r.r. form. Unique soln $\boxed{(15,-7)}$

d) Not in r.r. form. 1st nonzero entry in row 2 should be a 1

e) Not in r.r. form. Leading 1's are not down and to the right of each other.

f) Is in r.r. form. $0x+0y+0z=1$
$0=1$ FALSE $\boxed{\text{No Soln.}}$

g) Not in r.r. form. Rows of all 0's must be below all rows w/ nonzero entries.

4. For the next two word problems do the following:
   I) Define the variables that are used in setting up the system of equations.
   II) Set up the system of equations that represents this problem. III) Solve for the solution.
   IV) If the solution is parametric, then tell what restrictions should be placed on the parameter(s). Also give three specic solutions.

   (a) Fred, Bob, and George are avid collectors of baseball cards. Among the three of them, they have 924 cards. Bob has three times as many cards as Fred, and George has 100 more cards than Fred and Bob do combined. How many cards do each of the friends have?

Let x = the # of cards ~~F~~Fred has.
Let y = - - - - - Bob - -
Let z = - - - - George - - -

$x+y+z=924$
$y=3x$
$z=100+x+y$

$x+y+z=924$
$-3x+y=0$
$-x-y+z=100$

$$\left[\begin{array}{ccc|c}1&1&1&924\\-3&1&0&0\\-1&-1&1&100\end{array}\right] \xrightarrow{rref} \left[\begin{array}{ccc|c}1&0&0&103\\0&1&0&309\\0&0&1&512\end{array}\right]$$

$\boxed{\text{Fred has 103 cards, Bob 309, George 512}}$

   (b) In a laboratory experiment, a researcher wants to provide a rabbit with exactly 1000 units of vitamin A, exactly 1600 units of vitamin C and exactly 2400 units of vitamin E. The rabbit is fed a mixture of three foods. Each gram of food 1 contains 2 units of vitamin A, 3 units of vitamin C, and 5 units of vitamin E. Each gram of food 2 contains 4 units of vitamin A, 7 units of vitamin C, and 9 units of vitamin E. Each gram of food 3 contains 6 units of vitamin A, 10 units of vitamin C, and 14 units of vitamin E. How many grams of each food should the rabbit be fed?

Let x = the # of grams of food 1
Let y = - - - - - 2
Let z = - - - - 3

	food 1	food 2	food 3	req
Vit A	2	4	6	1000
Vit C	3	7	10	1600
Vit E	5	9	14	2400

$2x+4y+6z=1000$
$3x+7y+10z=1600$
$5x+9y+14z=2400$

$$\left[\begin{array}{ccc|c}2&4&6&1000\\3&7&10&1600\\5&9&14&2400\end{array}\right] \xrightarrow{rref} \begin{array}{cccc} x & y & z & \\ \end{array} \left[\begin{array}{ccc|c}1&0&1&300\\0&1&1&100\\0&0&0&0\end{array}\right]$$

Let z = t where $t \geq 0$.

$x+z=300$
$x=300-z$
$x=300-t$
$300-t\geq 0$
$-t\geq -300$
$t\leq 300$

$y+z=100$
$y=100-z$
$y=100-t$
$100-t\geq 0$
$-t\geq -100$
$t\leq 100$

The parametric soln is

$(300-t,\ 100-t,\ t)$, where $0\leq t\leq 100$.

5. Solve the system $\begin{aligned} 5x-3y &= 7 \\ 2x+6y &= -1 \end{aligned}$ using the Gauss-Jordan elimination method.

$$\left[\begin{array}{cc|c} 5 & -3 & 7 \\ 2 & 6 & -1 \end{array}\right] \xrightarrow{R_1-2R_2} \left[\begin{array}{cc|c} 1 & -15 & 9 \\ 2 & 6 & -1 \end{array}\right] \xrightarrow{R_2-2R_1} \left[\begin{array}{cc|c} 1 & -15 & 9 \\ 0 & 36 & -19 \end{array}\right]$$

$[5\ \ -3\ \ 7]-2[2\ \ 6\ \ -1]$
$=[5\ \ -3\ \ 7]+[-4\ \ -12\ \ 2]$
$=[1\ \ -15\ \ 9]$

$[2\ \ 6\ \ -1]-2[1\ \ -15\ \ 9]$
$=[2\ \ 6\ \ -1]+[-2\ \ 30\ \ -18]$
$=[0\ \ 36\ \ -19]$

$$\xrightarrow{\frac{1}{36}R_2} \left[\begin{array}{cc|c} 1 & -15 & 9 \\ 0 & 1 & \frac{-19}{36} \end{array}\right] \xrightarrow{R_1+15R_2} \left[\begin{array}{cc|c} 1 & 0 & 13/12 \\ 0 & 1 & -19/36 \end{array}\right]$$

$$\boxed{x=\frac{13}{12} \quad y=-\frac{19}{36}}$$

$[1\ \ -15\ \ 9]+15[0\ \ 1\ \ \frac{-19}{36}]$
$=[1\ \ -15\ \ 9]+[0\ \ 15\ \ \frac{-95}{12}]$
$=[1\ \ 0\ \ \frac{13}{12}]$

6. Solve for the variables *x*, *y*, *z*, and *u*. If this is not possible, explain why.

$$\begin{bmatrix} -1 & 0 & 5 \\ 7 & 3 & 0 \end{bmatrix}\begin{bmatrix} 7x & -2 \\ 4 & -3z \\ x & 8 \end{bmatrix}+\begin{bmatrix} y & -40x \\ -12 & 16 \end{bmatrix}^T=\begin{bmatrix} 9 & 3u \\ -6 & 0 \end{bmatrix}$$

$\underbrace{2\times 3 \quad 3\times 2}_{2\times 2}$

$$\begin{bmatrix} -7x+5x & 2+40 \\ 49x+12 & -14-9z \end{bmatrix}+\begin{bmatrix} y & -12 \\ -40x & 16 \end{bmatrix}=\begin{bmatrix} 9 & 3u \\ -6 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -2x & 42 \\ 49x+12 & -14-9z \end{bmatrix}+\begin{bmatrix} y & -12 \\ -40x & 16 \end{bmatrix}=$$

$$\begin{bmatrix} -2x+y & 42-12 \\ 49x+12-40x & -14-9z+16 \end{bmatrix}=\begin{bmatrix} 9 & 3u \\ -6 & 0 \end{bmatrix}$$

$\leftarrow -2x+y=9$

$-2(-2)+y=9$
$4+y=9$
$\boxed{y=5}$

$30=3u$
$\boxed{10=u}$

$9x+12=-6$
$9x=-18$
$\boxed{x=-2}$

$-9z+2=0$
$-9z=-2$
$\boxed{z=2/9}$

7. Find the matrix A that makes the following equation true:

$$\underbrace{\begin{bmatrix} -5 & 3 \\ 8 & 7 \end{bmatrix}}_{B} + \frac{1}{3}\underbrace{\begin{bmatrix} 2 & -1 \\ -7 & 5 \end{bmatrix}}_{C} A = I_2$$

$B + \frac{1}{3}CA = I_2$

Type B and C in calc.

$\frac{1}{3}CA = I_2 - B$

$C^{-1}CA = C^{-1}[3(I_2 - B)]$

$A = C^{-1}(3I_2 - 3B)$

$$\boxed{A = \begin{bmatrix} 22 & -21 \\ 26 & -33 \end{bmatrix}}$$

8. Use the given matrices to compute each of the following. If an operation is not possible, explain why.

$$A = \begin{bmatrix} -5 & 3 \\ 7 & 8 \end{bmatrix},\ B = \begin{bmatrix} 1 & 2 \\ 5 & -8 \\ 3 & 5 \end{bmatrix},\ C = \begin{bmatrix} 5 & 4 & 7 \\ 6 & -3 & 1 \end{bmatrix},$$ $D = 3 \times 3$ nonsingular matrix. $E = 4 \times 4$ singular matrix.

(a) $B+C$
(b) BC
(c) $D^{-1}C$
(d) $AB^T - 5C$
(e) DD^{-1}
(f) $E^{-1}E$
(g) CA
(h) C^{-1}

a) B is 3x2, C is 2x3 } dif. sizes, so cannot add.

b) size of B: 3x2, size of C: 2x3 ✓ → 3x3. BC is 3x3

c) size of D^{-1}: 3x3, size of C: 2x3, ≠ — not possible — the # of columns of D^{-1} ≠ # of rows of C

d) size of A: 2x2, size of B^T: 2x3 ✓ → 2x3; + (−5)C: 2x3 — same, so can subtract

$AB^T - 5C = \boxed{\begin{bmatrix} -24 & -69 & -35 \\ -7 & -14 & 56 \end{bmatrix}}$ (in calc)

e) $DD^{-1} = \boxed{I_3}$ — 3x3, 3x3 → 3x3

f) $E^{-1}E$ Not possible - E is singular, so E has no inverse

g) size of C: 2x3, size of A: 2x2, ≠ — Not possible - the # of columns of C ≠ the # of rows of A.

h) C^{-1} Not possible. Only square matrices can have inverses.

9. Work #41 on page 130 of Section 2.5 in your textbook by Tan. This is a problem about understanding the meaning of the entries in the product of two matrices. There is also another example in Week in Review #2.

10. First write the following system as a matrix equation and then solve by using a matrix inverse.

$$\begin{aligned} -3x &= 4y+z \\ y-7 &= -x+5z \\ 2x+z &= 14-y \end{aligned}$$

$-3x-4y-z=0$
$x+y-5z=7$
$2x+y+z=14$

The matrix eqn is $AX=B$ where

$A=\begin{bmatrix}-3 & -4 & -1\\ 1 & 1 & -5\\ 2 & 1 & 1\end{bmatrix}$, $X=\begin{bmatrix}x\\y\\z\end{bmatrix}$, and $B=\begin{bmatrix}0\\7\\14\end{bmatrix}$

$X=A^{-1}B=\begin{bmatrix}35/3\\-77/9\\-7/9\end{bmatrix}$ $\boxed{\begin{array}{l}x=35/3\\ y=-77/9\\ z=-7/9\end{array}}$

11. A simple economy depends on three commodities: oil, corn, and coffee. Production of 1 unit of oil requires 0.1 units of oil and 0.2 units of corn. Producing 1 unit of corn requires 0.2 units of oil, 0.1 units of corn, and 0.05 units of coffee. To produce 1 unit of coffee, 0.1 units of oil, 0.05 units of corn, and 0.1 units of coffee are used.

(a) Find the production level required to meet an external demand of 1,000 units of each of these three commodities. (#18, pg. 114 of *Finite Mathematics and Calculus* by Lial, Greenwell, and Ritchey)

Let x_1 = the # of units of oil produced
Let x_2 = the # of units of corn produced
Let x_3 = the # of units of coffee produced

Input-Output matrix

$$A=\begin{array}{c|ccc} & \text{oil} & \text{corn} & \text{coffee}\\ \hline \text{oil} & .1 & .2 & .1\\ \text{corn} & .2 & .1 & .05\\ \text{coffee} & 0 & .05 & .1\end{array}$$

$X=\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$, $D=\begin{bmatrix}1000\\1000\\1000\end{bmatrix}$

$X=(I_3-A)^{-1}D=\begin{bmatrix}1583.9072\\1529.5397\\1196.0855\end{bmatrix}$

$\boxed{\text{1,583.9072 units of oil, 1,529.5397 units of corn, 1,196.0855 units of coffee should be produced}}$

(b) How much of each commodity is consumed internally to meet this demand?

$\underbrace{X}_{\text{Total production}}-\underbrace{D}_{\text{demand}}=\underbrace{\begin{bmatrix}583.9072\\529.5397\\196.0855\end{bmatrix}}_{\text{Internal consumption}}$

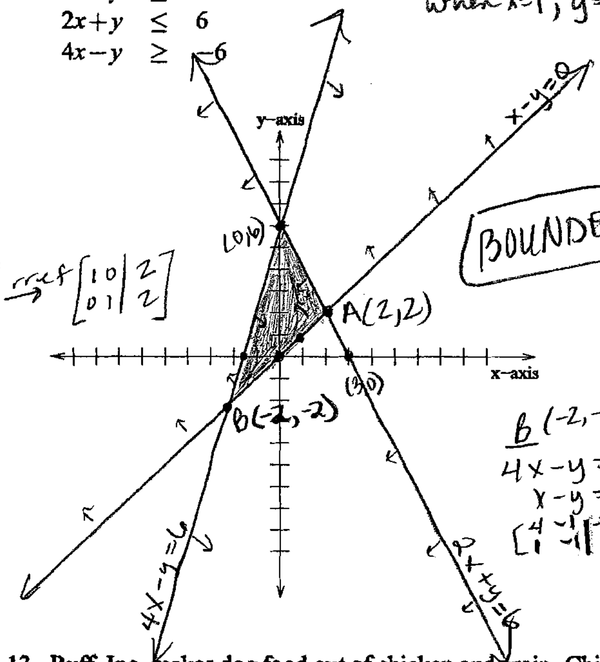
583.9072 units of oil, 529.5397 units of corn, and 196.0855 units of coffee are consumed.

12. **Graph the solution set for the following system of inequalities. Label all corner points. Is the solution set bounded or unbounded?**

$$\begin{aligned} x-y &\le 0 \\ 2x+y &\le 6 \\ 4x-y &\ge -6 \end{aligned}$$

$x-y=0$
when $x=0$, $y=0$ $(0,0)$
when $x=1$, $y=1$ $(1,1)$

Test $(0,5)$
$0-5 \le 0$?
$-5 \le 0$ TRUE



BOUNDED

A(2,2)
$2x+y=6$
$x-y=0$

$$\left[\begin{array}{cc|c} 2 & 1 & 6 \\ 1 & -1 & 0 \end{array}\right] \xrightarrow{rref} \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 2 \end{array}\right]$$

$(2,2)$

$2x+y=6$
$x=0$, $y=6$ $(0,6)$
$y=0$ $2x=6$
$x=3$ $(3,0)$

Test $(0,0)$
$2(0)+0 \le 6$?
$0 \le 6$ TRUE

B $(-2,-2)$
$4x-y=-6$
$x-y=0$

$$\left[\begin{array}{cc|c} 4 & -1 & -6 \\ 1 & -1 & 0 \end{array}\right] \xrightarrow{rref} \left[\begin{array}{cc|c} 1 & 0 & -2 \\ 0 & 1 & -2 \end{array}\right]$$

$(-2,-2)$

$4x-y=-6$
$x=0$ $y=6$ $(0,6)$
$y=0$ $4x=-6$
$x=-\frac{3}{2}$ $(-\frac{3}{2},0)$

Test $(0,0)$
$4(0)-0 \ge -6$?
$0 \ge -6$ TRUE

13. **Ruff, Inc. makes dog food out of chicken and grain. Chicken has 10 grams of protein and 5 grams of fat per ounce, and grain has 2 grams of protein and 2 grams of fat per ounce. A bag of dog food must contain at least 200 grams of protein and at least 150 grams of fat. If chicken costs 10 cents per ounce and grain costs 1 cent per ounce, how many ounces of each should Ruff use in each bag of dog food in order to minimize cost? Formulate as a linear programming problem, but do not solve.**

(#23, pg. 217 of *Finite Mathematics* by Waner and Costenoble)

Let x = the number of ounces of chicken that should be used in each bag.
Let y = the number of ounces of grain that should be used in each bag.

	Chicken	Grain	Min Req.
grams of protein per ounce	10g	2g	200 g
grams of fat per ounce	5g	2g	150g
cost per ounce	\$0.10	\$0.01	

Minimize Cost $C = 0.1x + 0.01y$

subject to
$$\begin{aligned} 10x+2y &\ge 200 \\ 5x+2y &\ge 150 \\ x &\ge 0 \\ y &\ge 0 \end{aligned}$$

14. A company makes two calculators: a business model and a scientific model. The business model contains 10 microcircuits and requires 20 minutes to program, while the scientific model contains 20 microcircuits and requires 30 minutes to program. The company has a contract that requires it to use at least 320 microcircuits each day, and the company has 14 hours of programming time available each day. The company also wants to make at least twice as many business calculators as scientific calculators. If each business calculator requires 10 production steps and each scientific calculator reqires 12 production steps, how many calculators of each type should be made each day to minimize the number of production steps? **Formulate as a linear programming problem, but do not solve.**

Let x = the number of business calculators to be made each day.
Let y = the number of scientific calculators to be made each day.

	Business	Scientific	
# of microcircuits	10	20	320 min req. per day
min. to program	20 min	30 min	14 hrs = 840 min of time avail.
# of production steps	10	12	

The # of business calc. should be at least 2 times the # of scientific calcs

$x \geq 2y$

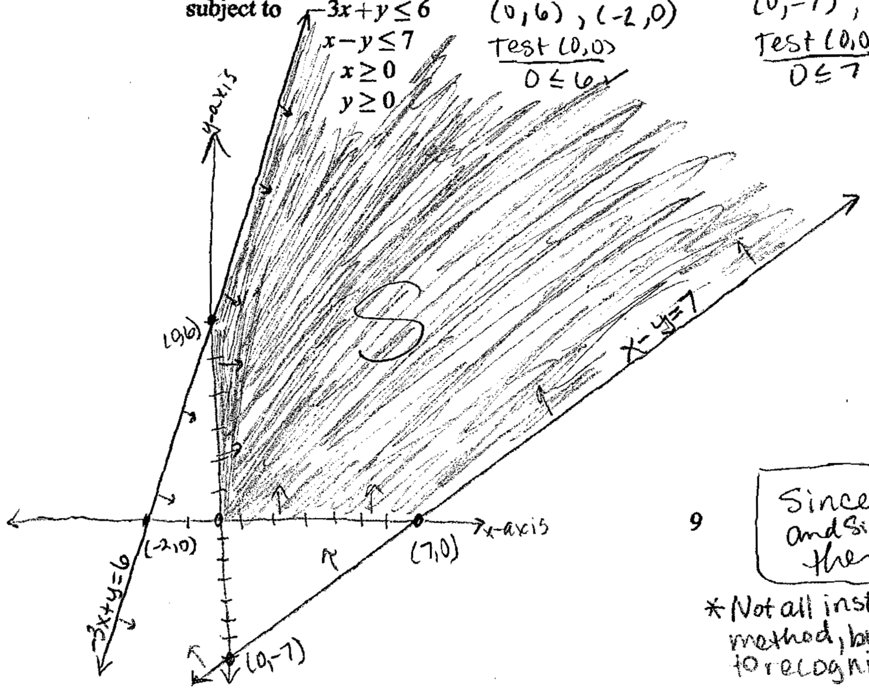
Minimize Number of Steps $N = 10x + 12y$
subject to
$$10x + 20y \geq 320$$
$$20x + 30y \leq 840$$
$$x \geq 2y$$
$$x \geq 0$$
$$y \geq 0$$

15. Solve using the Method of Corners.

Maximize $P = 2x + 5y$
subject to
$$-3x + y \leq 6$$
$$x - y \leq 7$$
$$x \geq 0$$
$$y \geq 0$$

① $-3x + y = 6$
$(0,6)$, $(-2,0)$
Test $(0,0)$
$0 \leq 6$ ✓

② $x - y = 7$
$(0,-7)$, $(7,0)$
Test $(0,0)$
$0 \leq 7$ ✓



Corners	Value of $P = 2x + 5y$
(0,6)	$5(6) = 30$
(0,0)	0
(7,0)	$2(7) = 14$
*Dummy corner (100,100)	$2(100) + 5(100) = 700$

Since S is unbounded and in quadrant I and since P has positive coefficients, there is no maximum.

*Not all instructors teach this method, but you should still be able to recognize that there is no maximum.

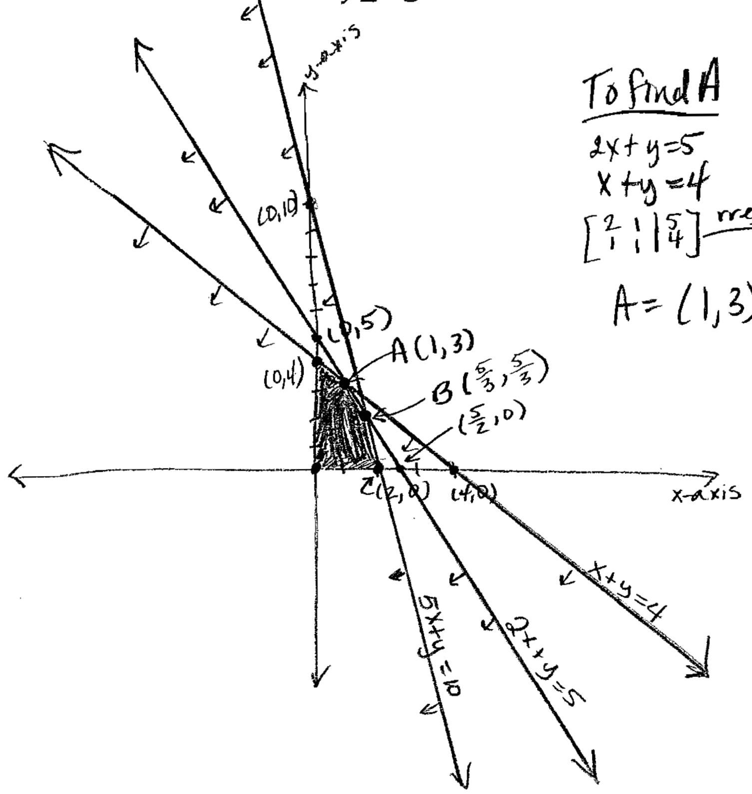
16. Solve using the Method of Corners.

Maximize $P = 2x + y$
subject to
$x + y \le 4$ } (1,3)
(1.5,2.5) $2x + y \le 5$ }
$5x + y \le 10$ $\left(\frac{5}{3}, \frac{5}{3}\right)$
$x \ge 0$
$y \ge 0$ } QI

① $x + y = 4$
(0,4) (4,0)
Test (0,0)
$0 \le 4$ ✓

② $2x + y = 5$
(0,5) $\left(\frac{5}{2}, 0\right)$
Test (0,0)
$0 \le 5$ ✓

③ $5x + y = 10$
When $x = 0$, $y = 10$ (0,10)
When $y = 0$, $5x = 10$ (2,0)
$x = 2$
Test (0,0)
$0 \le 10$ ✓



To find A

$2x + y = 5$
$x + y = 4$

$\left[\begin{array}{cc|c} 2 & 1 & 5 \\ 1 & 1 & 4 \end{array}\right] \xrightarrow{rref} \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 3 \end{array}\right]$

$A = (1, 3)$

To find B

$5x + y = 10$
$2x + y = 5$

$\left[\begin{array}{cc|c} 5 & 1 & 10 \\ 2 & 1 & 5 \end{array}\right] \xrightarrow{rref} \left[\begin{array}{cc|c} 1 & 0 & 5/3 \\ 0 & 1 & 5/3 \end{array}\right]$

$B = \left(\frac{5}{3}, \frac{5}{3}\right)$

Corners	Value of $P = 2x + y$
(0,4)	$2(0) + 4 = 4$
(0,0)	0
(2,0)	$2(2) + 0 = 4$
(1,3)	$2(1) + 3 = 5$ ←
$\left(\frac{5}{3}, \frac{5}{3}\right)$	$2\left(\frac{5}{3}\right) + \frac{5}{3} = \frac{10}{3} + \frac{5}{3} = \frac{15}{3} = 5$ ←

Conclusion: The maximum value of P is 5 and occurs at every point on the line segment connecting (1,3) to $\left(\frac{5}{3}, \frac{5}{3}\right)$. There are infinitely many solutions.